Our normal modes are

$$a_1 = \frac{1}{\sqrt{3m}}\begin{pmatrix}1\\1\\1\end{pmatrix} \quad a_2 = \frac{1}{\sqrt{2m}}\begin{pmatrix}1\\0\\-1\end{pmatrix} \quad a_3 = \frac{1}{\sqrt{6m}}\begin{pmatrix}1\\-2\\1\end{pmatrix}$$

We use these vectors as a basis set to set to write an arbitrary displacement as $\quad \eta(t) = \xi_1\vec{a}_1 + \xi_2\vec{a}_2 + \xi_3\vec{a}_3$

Suppose we apply a force $F(t)$, $\Rightarrow$

$$\overleftrightarrow{T}\left(\sum_{i=1}^{3}\ddot{\xi}_i\vec{a}_i\right) + \overleftrightarrow{V}\left(\sum_{i=1}^{3}\xi_i\vec{a}_i\right) = \vec{F}(t)$$

$$\Rightarrow \quad \ddot{\xi}_j + \omega_j^2\xi_j = f_j(t) \qquad \text{where } f_j = \vec{a}_j^{\,T}F(t)$$

in this problem. $\vec{F} = \begin{pmatrix} f\cos\omega t\\ 0\\ 0\end{pmatrix}$

$$\therefore \; f_1 = \frac{1}{\sqrt{3m}} f\cos\omega t \qquad f_2 = \frac{1}{\sqrt{2m}} f\cos\omega t \qquad f_3 = \frac{1}{\sqrt{6m}} f\cos\omega t .$$

with initial conditions $\dot{\xi}_i = 0 \quad \xi_i = 0$

$$\Rightarrow \quad \xi_1 = \frac{f}{\sqrt{3m}\,\omega^2}(1-\cos\omega t) \qquad \xi_2 = \frac{f}{\sqrt{2m}\,(\omega_2^2-\omega^2)}(\cos\omega t - \cos\omega_2 t).$$

$$\xi_3 = \frac{f}{\sqrt{6m}\,(\omega_3^2-\omega^2)}(\cos\omega t - \cos\omega_3 t).$$

$$\Rightarrow \eta_3(t) = \frac{f}{m}\left[\frac{1}{3\omega^2}(1-\cos\omega t) - \frac{1}{2(\omega_2^2-\omega^2)}(\cos\omega t - \cos\omega_2 t) + \frac{1}{6(\omega_3^2-\omega^2)}(\cos\omega t - \cos\omega_3 t)\right].$$