To find the relative amplitudes of $x_1 + x_2$ (the mode shape), divide $x_1(t)$ by $x_2(t)$.

$$\frac{x_1(t)}{x_2(t)} = \frac{A}{B} \quad \text{for both frequencies.}$$

From the simplified system equations (1),

$$\frac{A}{B} = \frac{k_c}{m\omega^2 - bj\omega - (k + k_c)}$$

$\frac{A}{B}$ for ω_1 is

$$\frac{A}{B} = \frac{k_c}{m\,(\omega_1^2) - bj\omega_1 - (k + k_c)}$$

$$\frac{A}{B} = \frac{k_c}{m\dfrac{(bj \pm \sqrt{-b^2 + 4km})^2}{(2m)^2} - bj\dfrac{(bj \pm \sqrt{-b^2 + 4km})}{2m} - (k+k_c)}$$

$$= \frac{k_c}{\frac{1}{4}\dfrac{(-b^2 \pm 2bj\sqrt{-b^2+4km} + -b^2+4km)}{m} + \dfrac{b^2}{2m}\dfrac{\pm bj\sqrt{-b^2+4km}}{2m} - (k+k_c)}$$

$$= \frac{k_c}{\dfrac{-b^2}{2m} + k - k - k_c}$$

$$= \frac{k_c}{\dfrac{-b^2}{2m} - k_c}$$

In most physical systems b is small and $\frac{A}{B} \approx -1$.

$\frac{A}{B}$ for ω_2 is:

$$\frac{A}{B} = \frac{k_c}{m\dfrac{(bj \pm \sqrt{-b^2+4(k+2k_c)m})^2}{2m} - bj\dfrac{(bj \pm \sqrt{-b^2+4(k+2k_c)m})}{2m} - k - k_c}$$

$$\frac{A}{B} = \frac{k_c}{\dfrac{1}{4m}\,(-b^2 \pm 2bj\sqrt{b^2+4(k+2k_c)m} - b^2 + 4(k+2k_c)m) + \dfrac{b^2}{2m} \pm \dfrac{bj\sqrt{b^2+4(k+2k_cm)}}{2m} - k - k_c}$$

$$\frac{A}{B} = \frac{k_c}{(k + 2k_c) - \dfrac{b^2}{2m} - (k + k_c)} = \frac{k_c}{\dfrac{-b^2}{2m} + k_c} .$$