5

$$[(k + k_c + bj\omega - m\omega^2) - k_c]$$

$$[k + k_c + bj\omega - m\omega^2) + k_c] = 0$$

or $$[m\omega^2 - k - bj\omega][m\omega^2 - k - 2k_c - bj\omega] = 0.$$

Either term may equal zero for the equation to be zero; therefore,

$$m\omega^2 - k - bj\omega = 0$$

will supply one root and

$$m\omega^2 - k - 2k_c - bj\omega = 0$$

will yield the second characteristic frequency.

Solving these equations using the quadratic formula,

$$\omega_1 = \frac{bj \pm \sqrt{-b^2 - 4km}}{2m}$$

$$\omega_2 = \frac{bj \pm \sqrt{-b^2 + 4(k + 2k_c)m}}{2m} .$$

$x_1(t)$ then is

$$Ae^{j\frac{(bj \pm \sqrt{-b^2 + 4km})t}{2m}}$$ and by Euler's theorem,

$$x_1(t) = Ae^{-bt/2m} \cos\left(\frac{\sqrt{-b^2 + 4km}}{2m}t + \theta\right).$$

Here θ is a phase shift angle determined by initial conditions.

Similarly,

$$x_2(t) = Be^{-bt/2m} \cos\left(\frac{\sqrt{-b^2 + 4km}}{2m} + \theta\right)$$

in the first mode (first characteristic frequency).

In the second mode,

$$x_1(t) = Ce^{-bt/2m} \cos\left(\frac{\sqrt{-b^2 + 4(k + k_c)m}}{2m} t + \theta\right)$$

$$x_2(t) = De^{-bt/2m} \cos\left(\frac{\sqrt{-b^2 + 4(k + k_c)m}}{2m} t + \theta\right)$$