5. $t=0 \quad x=0.$

a) equation of motion of the platform : $m\ddot{x} + \gamma\dot{x} + kx + mg = 0$

b) let $y = x + mg/k$ and write $\gamma = 2m\alpha$. $\Rightarrow$

$$\ddot{y} + 2\alpha\dot{y} + \frac{k}{m}y = 0.$$

general solution : $y = Ae^{\omega_+ t} + B^{\omega_- t}$ $\qquad \omega_\pm = -\alpha \pm \sqrt{\alpha^2 - k/m}$

Criticlal damping occurs when the expression in the radical vanishes, $\alpha^2 = k/m$.

$\therefore \omega_\pm = -\alpha.$

and general solution is of a different form: $y = (A + Bt)e^{-\alpha t}$ $\quad *$

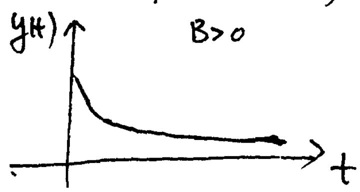
If we start measuring time from the moment the putty hits the platform, then the Initial conditions are $y(0) = A = \frac{mg}{k} = g/\alpha^2$

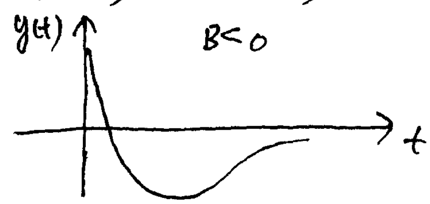
$$\dot{y}(0) = (B - \alpha A) = -\sqrt{2gh}$$

$$\Rightarrow y(t) = \left[\frac{g}{\alpha^2} + \left(\frac{g}{\alpha} - \sqrt{2gh}\right)t\right]e^{-\alpha t}$$

Note that the sign of B, the coefficient of t in equation $*$, determines whether there will be overshoot. If $B>0$, (i.e. $k/m < g/2h$), then there is no overshoot; $B \not< 0$, for large t, $y(t)<0$, so overshoot occurs.

c)





d) Suppose now the system is overdamped, i.e. $\alpha^2 > k/m$, and write

$$\omega_\pm = -\alpha \pm \sqrt{\alpha^2 - k/m}$$

note : $|\omega_-| > \alpha > |\omega_+|$, general overdamped solution $y(t) = Ae^{\omega_+ t} + Be^{\omega_- t}$

for ω_+ vanish, the initial condition are

$$y(0) = A + B = mg/k$$

$$y'(0) = A\omega_+ + B\omega_- = -\sqrt{2gh}$$

set $A=0$ $\Rightarrow$ $\left.\begin{array}{l} \omega_- = -\sqrt{2gh}\,\frac{k}{mg} = -\alpha - \sqrt{\alpha^2 - k/m} \\ \gamma = 2m\alpha \end{array}\right\} \Rightarrow \gamma = m\sqrt{\frac{g}{2h}} + k\sqrt{\frac{2h}{g}}$