2. * $2y''-12y'+10y=0$

find roots of $2r^2-12r+10=0 \Rightarrow (r-1)(2r-10)=0$

$\therefore r_1=1 \quad r_2=5$

$\therefore y=C_1e^{5t}+C_2e^{t}$

* $4y''-12y'+9y=0$

find roots of $4r^2-12r+9=0 \Rightarrow (2r-3)^2=0 \Rightarrow r_{1,2}=3/2$

$\therefore y=C_1te^{3/2\,t}+C_2e^{3/2\,t}$

* $y''+2y'+5y=0$

find roots of $r^2+2r+5=0 \Rightarrow r_1=-1+i2 \quad r_2=-1-2i.$

$\therefore y=C_1e^{-t}e^{i2t}+C_2e^{-t}e^{-2it}.$

$Re(y)=e^{-t}(C_1\cos 2t+C_2\cos 2t)=Ce^{-t}\cos 2t.$

4. equation: $qE_0\cos\omega t+\alpha x'''=mx''$

let $x=C_1\cos\omega t+C_2\sin\omega t.$

$\therefore x'=-C_1\omega\sin\omega t+C_2\omega\cos\omega t$

$x''=-C_1\omega^2\cos\omega t-C_2\omega^2\sin\omega t$

$x'''=C_1\omega^3\sin\omega t-C_2\omega^3\cos\omega t$

$\therefore qE_0\cos\omega t+\alpha C_1\omega^3\sin\omega t-\alpha C_2\omega^3\cos\omega t=-mC_1\omega^2\cos\omega t-mC_2\omega^2\sin\omega t$

equal the coefficient before $\sin\omega t$ and $\cos\omega t$

$qE_0-\alpha C_2\omega^3=-mC_1\omega^2$

$\alpha C_1\omega^3=-mC_2\omega^2 \Rightarrow \frac{C_1}{C_2}=-\frac{m}{\alpha\omega}.$

$C_2=qE_0/(\alpha\omega^3+m^2\omega/\alpha)$