Solution to HW3

1. a) $y''+4y=0 \Rightarrow \omega^2=4 \quad \therefore \omega=2$

let $y = c_1\sin 2t + c_2\cos 2t$

* $y(0)=0 \Rightarrow c_2=0$

$y(\pi/4)=1 \Rightarrow c_1\sin(2\cdot\frac{\pi}{4}) + c_2\cos(2\cdot\frac{\pi}{4}) = 1 \Rightarrow c_1=1$

$\therefore \boxed{y=\sin 2t}$

* $y(\pi/2)=-1 \quad y'(\pi/2)=1 \Rightarrow \left.\begin{array}{l} c_1\sin(2\cdot\frac{\pi}{2}) + c_2\cos(2\cdot\frac{\pi}{2}) = -1 \\ y'=2c_1\cos 2t - 2c_2\sin 2t \Rightarrow 2c_1\cos(2\cdot\frac{\pi}{2}) - 2c_2\sin(2\cdot\frac{\pi}{2}) = 1 \end{array}\right\}$

$\Rightarrow c_2=1 \quad c_1=-\frac{1}{2}$

$\therefore y = \cos 2t - \frac{1}{2}\sin 2t$

* similiarly $\left.\begin{array}{l} y(0)=0 \\ y'(0)=1 \end{array}\right\} \Rightarrow y = \sin 2t/2$

* $\left.\begin{array}{l} y(\pi/4)=a \\ y''(0)=b \end{array}\right\} \Rightarrow \left\{\begin{array}{l} c_1 = -b/4 \\ c_2 = a \end{array}\right.$

$\therefore y = a\sin 2t - \frac{b}{4}\cos 2t$.

b) $y''+y=2y' \Rightarrow y''-2y'+y=0$

find the roots of $r^2-2r+1=0 \Rightarrow r=1$

$\therefore y = c_1te^t + c_2e^t$

$y(0)=1 \Rightarrow c_2=1$

$y(1)=1 \Rightarrow c_1e + c_2e = 1 \Rightarrow c_1 = \frac{1-e}{e}$

$\therefore y = (\frac{1-e}{e})te^t + e^t$