Energy can have the unit of kinetic energy : $mv^2 = \left[\frac{\text{mass} \cdot \text{Distance}^2}{\text{Time}^2}\right]$

$\therefore$ k unit is $\left[\frac{\text{mass}}{\text{Time}^2}\right]$, Period T has the time unit, therefor,

let's try $\sqrt{\frac{m}{k}}$, the unit is $\left[\text{mass} \Big/ \frac{\text{mass}}{\text{Time}^2}\right]^{1/2} = \text{Time}.$

$\therefore$ Period $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

3. let's $m=1$, $k=1$, $x_m=1$, $a=1$

$\therefore V(x) = \frac{1}{2}x^2 + x^4 \qquad E = \frac{1}{2}kx_m^2 + ax_m^4 = \frac{3}{2}$

$\therefore T = \sqrt{8}\int_0^1 \frac{dx}{(\frac{3}{2}-\frac{1}{2}x^2-x^4)^{1/2}} = \frac{4}{\sqrt{3}}\int_0^1 dx \Big/ (1-\frac{1}{3}x^2-\frac{2}{3}x^4)^{1/2}$

$\because x<1, \quad \therefore x^4 << x^2 << 1, \quad \therefore \frac{1}{3}x^2 + \frac{2}{3}x^4 < 1$

$\therefore T \approx \frac{4}{\sqrt{3}}\int_0^1 dx\,(1-\frac{1}{3}x^2-\frac{2}{3}x^4)$ - ← taylor expansion.

$\doteq 1.8$

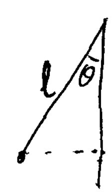
Dimensional Analysis: a unit is $\left[\frac{\text{Energy}}{\text{Distance}^4}\right] = \left[\frac{\text{mass} \cdot \text{Distance}^2}{\text{Distance}^4\ \text{Time}^2}\right] = \left[\frac{\text{mass}}{\text{Distance}^2\ \text{time}^2}\right]$

$\therefore$ let's try combination $\frac{1}{x_m}\sqrt{\frac{m}{a}}$, unit is time.

$\therefore \boxed{T \doteq 1.8\left(\sqrt{\frac{m}{k}} + \frac{1}{x_m}\sqrt{\frac{m}{a}}\right)}$

4. $V(\theta) = mgl(1-\cos\theta)$

$\cos\theta = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} - \cdots$



To lowerst order $V(\theta) \doteq \frac{1}{2}mgl\theta^2. = \frac{1}{2}\frac{mg}{l}(l\theta)^2$

$l\theta \sim x. \quad \frac{mg}{l} \sim k$

$\therefore T = 2\pi\sqrt{\frac{l}{g}}$

for anharmonic correction. $V(\theta) \doteq \frac{1}{2}mgl\left(\theta^2 - \frac{\theta^4}{24}\right) = \frac{1}{2}\frac{mg}{l}(l\theta)^2 - \frac{1}{48}\frac{mg}{l^3}(l\theta)^4$

$\therefore$ ~~$T = 1.8(\sqrt{\frac{g}{l}}$~~ $\quad a \sim \frac{1}{48}\frac{mg}{l^3} \quad \therefore T \propto \sqrt{\frac{l}{g}} + \frac{1}{l\theta_m}\sqrt{\frac{m}{a}} = \sqrt{\frac{l}{g}} + \frac{4\sqrt{3}}{\theta_m}\sqrt{\frac{l}{g}}$

rate of correction is: $R = \theta_m / 4\sqrt{3} \quad R = 1\% \quad \theta_m = 4\sqrt{3}/100 \quad R = 10/100 \quad \theta_m = 4\sqrt{3}/10$