Problem for HONORS:

1. The time dt it takes the particle to travel a distance dx is

$$dt = \frac{dx}{v} \qquad v = \frac{dx}{dt}.$$

∴ The Period of an oscillation with total energy E and amplitude x_m is

$$T = 4\int_0^{x_m(t)} dt' = 4\int_0^{x_m} \frac{dx}{v}$$

From energy conservation $\frac{1}{2}mv^2 + V = E = \text{constant} = \frac{1}{2}kx_m^2 + ax_m^4$

$$\therefore v = \sqrt{\frac{2}{m}(E-V)}$$

$$\therefore T = 4\int_0^{x_m} \frac{dx}{\sqrt{\frac{2}{m}}\sqrt{E-V}} = \sqrt{8m}\int_0^{x_m} \frac{dx}{\sqrt{E-V}}$$

□

2. for $a=0$, $\quad V(x) = \frac{1}{2}kx^2 \quad E = \frac{1}{2}kx_m^2$

let $m=1$, $k=1$ and $x_m=1$

$$\therefore T = \sqrt{8}\int_0^1 \frac{dx}{\sqrt{E-V}} \qquad E = \frac{1}{2} \quad V = \frac{x^2}{2}$$

$$= \sqrt{8}\int_0^1 \frac{dx}{\sqrt{\frac{1}{2}-\frac{x^2}{2}}} \quad \cdots = 4\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

let $x = \cos\theta$ $\quad \therefore dx = +d(\cos\theta) = -\sin\theta\, d\theta$

$$\sqrt{1-x^2} = \sin\theta$$

and $0 \le \theta \le \frac{\pi}{2}$

$$\therefore T = 4\int_{\frac{\pi}{2}}^{0} (-d\theta) = 2\pi$$

Dimensional analysis: let's see $V(x) = \frac{1}{2}kx^2$

$\therefore$ k unit is $\left[\frac{\text{Energy}}{\text{Distance}^2}\right]$