5. a) $1^2 = e^{2\ln(1)} = e^{2(i2\pi n)} = e^{i4\pi n} = \cos(4n\pi) + i\,\cancel{\sin(4n\pi)}^{\to 0} = 1$ for all n

b) $1^i = e^{i\ln(1)} = e^{i(i2\pi n)} = e^{-2n\pi}$ $\quad n = 0, \pm 1 \dots$

c) $i^i = e^{i\ln i} = e^{i[\ln(1) + i(\frac{\pi}{2} + 2n\pi)]} = e^{-\frac{\pi}{2}(1+4n)}$ $\quad n = 0, \pm 1 \dots$

6. $z = \mathrm{Re}[A e^{i(\omega t + \alpha)}] = A\cos(\omega t + \alpha)$

$= A\cos\omega t\cos\alpha - A\sin\omega t\sin\alpha$

$= (A\cos\alpha)\cos\omega t + (-A\sin\alpha)\sin\omega t$

a) $z = \sin\omega t + \cos\omega t$

comparing: $A\cos\alpha = 1$ and $-A\sin\alpha = 1$

$\Rightarrow A^2\cos^2\alpha + A^2\sin^2\alpha = 2 \Rightarrow A = \sqrt{2}$

$\Rightarrow \tan\alpha = \frac{-1}{1} = -1 \quad \because -\frac{\pi}{2} \le \alpha \le 0 \Rightarrow \alpha = -\frac{\pi}{4}$

$\therefore z = \mathrm{Re}[\sqrt{2}\, e^{i(\omega t - \frac{\pi}{4})}]$

b) $z = \cos(\omega t - \pi/3) - \cos\omega t$ $\quad \sqrt{3}/2$

$= \cos\omega t\, \cancel{\cos\frac{\pi}{3}}^{\to 1/2} - \sin\omega t\, \cancel{\sin\frac{\pi}{3}} - \cos(\omega t)$

$= -\frac{1}{2}\cos(\omega t) - \frac{\sqrt{3}}{2}\sin(\omega t)$

$\begin{cases} A\cos\alpha = -1/2 \\ A\sin\alpha = -\sqrt{3}/2 \end{cases} \Rightarrow A^2 = \frac{1}{4} + \frac{3}{4} = 1 \quad \boxed{A = 1}$

$\alpha = \tan^{-1}\left(\frac{-\sqrt{3}/2}{-1/2}\right) \qquad -\pi \le \alpha \le -\frac{\pi}{2}$

$\boxed{\alpha = \frac{\pi}{3} - \pi = -\frac{2}{3}\pi}$

Remember that the function $\tan^{-1}(x)$ doesn't give you the final answer always, you must use the information together with the data of what quadrant is the angle located in.