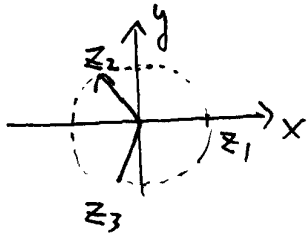


2. ii)



$$3. a) (z_1 + z_2)^* = [(x_1 + iy_1) + (x_2 + iy_2)]^* = [(x_1 + x_2) + i(y_1 + y_2)]^* \\ = (x_1 + x_2) - i(y_1 + y_2) = (x_1 - iy_1) + (x_2 - iy_2) = z_1^* + z_2^*$$

$$b) (z_1 z_2)^* = [(x_1 + iy_1)(x_2 + iy_2)]^* = [(x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)]^* \\ = (x_1 x_2 - y_1 y_2) - i(x_1 y_2 + x_2 y_1) = (x_1 - iy_1)(x_2 - iy_2) = z_1^* z_2^*$$

$$c) \left(\frac{1}{z}\right)^* = \left(\frac{1}{x+iy}\right)^* = \left(\frac{x-iy}{x^2-y^2}\right)^* = \frac{x+iy}{x^2-y^2} = \frac{x+iy}{(x+iy)(x-iy)} = \frac{1}{x-iy} = \frac{1}{z^*}$$

$$d) \left(\frac{z_1}{z_2}\right)^* = \left(z_1 \cdot \frac{1}{z_2}\right)^* = z_1^* \cdot \left(\frac{1}{z_2}\right)^* = z_1^* \cdot \frac{1}{z_2^*} = \frac{z_1^*}{z_2^*}$$

$$e) (e^z)^* = (e^{x+iy})^* = [e^x \cos y + i e^x \sin y]^* = e^x \cos y - i e^x \sin y = e^{x-iy} = e^{z^*}$$

4.

$$a) \ln(1) = \ln(e^{i2\pi n}) \quad n = 0, \pm 1, \pm 2$$

$$\ln(1) = i2\pi n$$

b), c). Any complex number can be expressed in polar form

$$z = r e^{i\theta}$$

$$\ln(z) = \ln(r e^{i(\theta + 2h\pi)})$$

$$h = 0, \pm 1, \pm 2$$

$$= \ln r + \ln(e^{i(\theta + 2h\pi)})$$

$$\boxed{\ln z = \ln r + i(\theta + 2h\pi)}$$