Again:

$$L\ddot{q} + R\dot{q} + \frac{1}{C} q = V_0 e^{i\Omega t}$$

Taking the derivative with respect to time we obtain.

$$L\ddot{I} + R\dot{I} + \frac{1}{C} I = i\Omega V_0 e^{i\Omega t}$$

Insert now the ansatz: $\boxed{I = I_0 e^{i(\Omega t - \varphi)}}$ into this differential eqn.

We obtain

$$\boxed{\left[R + i\left(\Omega L - \frac{1}{\Omega C}\right)\right] I_0 = V_0 e^{i\varphi} = V_0(\cos\varphi + i\sin\varphi)} \quad (*)$$

From the absolute value of this complex number we obtain the value of I_0:

$$\left[R^2 + \left(\Omega L - \frac{1}{\Omega C}\right)^2\right] I_0^2 = V_0^2 \Rightarrow$$

Taking square root: $\boxed{I_0 = \frac{V_0}{\sqrt{\left(\Omega L - \frac{1}{\Omega C}\right)^2 + R^2}}}$