

The most general solution to the inhomogeneous eqn takes therefore the form:

$$x_1(t) = C_1 \sin \omega_0 t + C_2 \cos \omega_0 t + ut.$$

The constants  $C_1, C_2$  are fixed by initial conditions.

$$x_1(t=0) = 0 \quad ; \quad \dot{x}_1(t=0) = 0.$$

From  $x_1(t=0) = 0$  we obtain  $C_2 = 0.$

From  $\dot{x}_1(t=0) = 0$  we obtain

$$\dot{x}_1(t) = \omega_0 C_1 \cos \omega_0 t + u.$$

At  $t=0$  we get.

$$0 = \omega_0 C_1 + u \Rightarrow C_1 = -\frac{u}{\omega_0}$$

So the final answer is:

$$x_1(t) = u \left( t - \frac{1}{\omega_0} \sin \omega_0 t \right)$$