HW# 2 -Phys273-Spring 2003

Due Friday, Feb. 14, 2003, by 9am

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1. Express the following in polar form $re^{i\theta}$: -6, $-5i$, $(1+i)/\sqrt{2}$, $(1-i)/\sqrt{2}$, $2+3i$, $(2+i)/(1+2i)$. (5 points)

2. (i) Find all the cube roots of unity $1^{1/3}$, and express them both in polar form and in the form $x+iy$ (with x and y real). (ii) Plot them in the complex plane. (5 points).

3. Show that the complex conjugate operation that sends $zz = x + iy$ to $z^* = x - iy$ enjoys the following properties: (10 points).

   (a) $(z_1 + z_2)^* = z_1^* + z_2^*$

   (b) $(z_1 z_2)^* = z_1^* z_2^*$

   (c) $(1/z)^* = 1/z^*$

   (d) $(z_1/z_2)^* = z_1^*/z_2^*$

   (e) $(e^z)^* = e^{z^*}$

4. The natural logarithm $\ln z$ of a complex number z is defined by the equation

$$e^{\ln z} = z.$$

   This equation does not determine $\ln z$ uniquely, so $\ln z$ is said to be a *multi-valued* function of z. (5 points).

   (a) Find all the values of $\ln 1$.

   (b) Show that for any z there are infinitely many values of $\ln z$.

   (c) Express $\ln(re^{i\theta})$ in terms of r and θ.

   (d) Show that $z^* = e^{(\ln z)^*}$.

5. Using the logarithm we can define z^w for any two complex numbers z and w, by

$$z^w = e^{w \ln z}.$$

   Find all the values of the following: (5 points)

   (a) 1^2

   (b) 1^i

   (c) $(-1)^i$

   (d) i^i

6. Express the following in the form $\mathrm{Re}\left[Ae^{i(\omega t+\phi)}\right]$: (10 points).

   (a) $\sin\omega t + \cos\omega t$

   (b) $\cos(\omega t - \pi/3) - \cos\omega t$

   (c) $2\sin\omega t + 3\cos\omega t$

   (d) $\sin\omega t - 2\cos(\omega t - \pi/4) + \cos\omega t$