

Physics 273, Fall 2006
Final Exam

Name

Closed-book examination. One pre-prepared 8 1/2" x 11" sheet of notes may be used. Calculator may be used. Show work for partial credit. Include UNITS on all answers. Problems are 30 points apiece. Total will be normalized to 100.

EXAM ADVICE: If you don't immediately see what to do on one problem, go on to the next problem and come back to the unfinished problems later.

TABLE D-5

Integration Formulas†

1. $\int A dt = At$
2. $\int At dt = \frac{1}{2}At^2$
3. $\int At^n dt = A \frac{t^{n+1}}{n+1} \quad n \neq -1$
4. $\int At^{-1} dt = A \ln t$
5. $\int e^{bt} dt = \frac{1}{b}e^{bt}$
6. $\int \cos \omega t dt = \frac{1}{\omega} \sin \omega t$
7. $\int \sin \omega t dt = -\frac{1}{\omega} \cos \omega t$
8. $\int_0^{\infty} e^{-ax} dx = \frac{1}{a}$
9. $\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$
10. $\int_0^{\infty} xe^{-ax^2} dx = \frac{2}{a}$
11. $\int_0^{\infty} x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}}$
12. $\int_0^{\infty} x^3 e^{-ax^2} dx = \frac{4}{a^2}$
13. $\int_0^{\infty} x^4 e^{-ax^2} dx = \frac{3}{8} \sqrt{\frac{\pi}{a^5}}$

TABLE D-3

Trigonometric Formulas

$$\sin^2 \theta + \cos^2 \theta = 1; \quad \sec^2 \theta - \tan^2 \theta = 1; \quad \csc^2 \theta - \cot^2 \theta = 1$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin \frac{1}{2} \theta = \sqrt{\frac{1 - \cos \theta}{2}}; \quad \cos \frac{1}{2} \theta = \sqrt{\frac{1 + \cos \theta}{2}}; \quad \tan \frac{1}{2} \theta = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A \pm \sin B = 2 \sin\left[\frac{1}{2}(A \pm B)\right] \cos\left[\frac{1}{2}(A \pm B)\right]$$

$$\cos A + \cos B = 2 \cos\left[\frac{1}{2}(A + B)\right] \cos\left[\frac{1}{2}(A - B)\right]$$

$$\cos A - \cos B = 2 \sin\left[\frac{1}{2}(A + B)\right] \sin\left[\frac{1}{2}(B - A)\right]$$

$$\tan A \pm \tan B = \frac{\sin(A \pm B)}{\cos A \cos B}$$

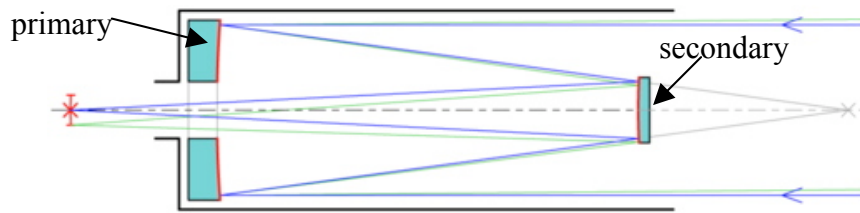
1. A thin film with an index of refraction 1.5 is surrounded by air. It is illuminated normally by white light and is viewed in reflection. The reflected light is missing wavelengths 360, 450 and 602 nm in the visible.

a) What are the wavelengths inside the film of the missing wavelengths?

b) What is the thickness of the film?

b) If the film were lying on a glass slide with index of refraction $n = 1.6$, find two wavelengths in the visible would be missing from the spectrum of reflected light.

2. The Hubble telescope has a collector mirror of diameter 2.4 m. It is a Cassegrain design (see diagram), with a concave collecting (primary) mirror and a convex secondary mirror separated by 4.907m. The focal point of the telescope is 57.6 m from the secondary mirror.



- a) Assuming the collecting mirror limits the resolution, what is the best angular resolution that could be obtained with the microscope for infrared light of wavelength 900 nm?
- b) For a detector plate of diameter 0.23 m, what is the light gathering power of the microscope?
- c) The radius of the curvature of the first mirror is 11.042 m. Assuming the mirrors are spherical (this isn't correct, they're really hyperbolic), what is the radius of curvature of the second mirror?

3. An entrepreneur has a plan to place a solar panel in space and send the energy down to earth in the form of electromagnetic waves. (There are really people working on this idea.) The collector on earth can't be too big, so the plan is to place a regularly spaced array of antennas in space and use interference to localize the energy onto the collector. The height of the array above the earth's surface will be 3.57×10^7 m (geosynchronous orbit). The chosen frequency of the electromagnetic radiation is 5×10^9 Hz (microwaves).

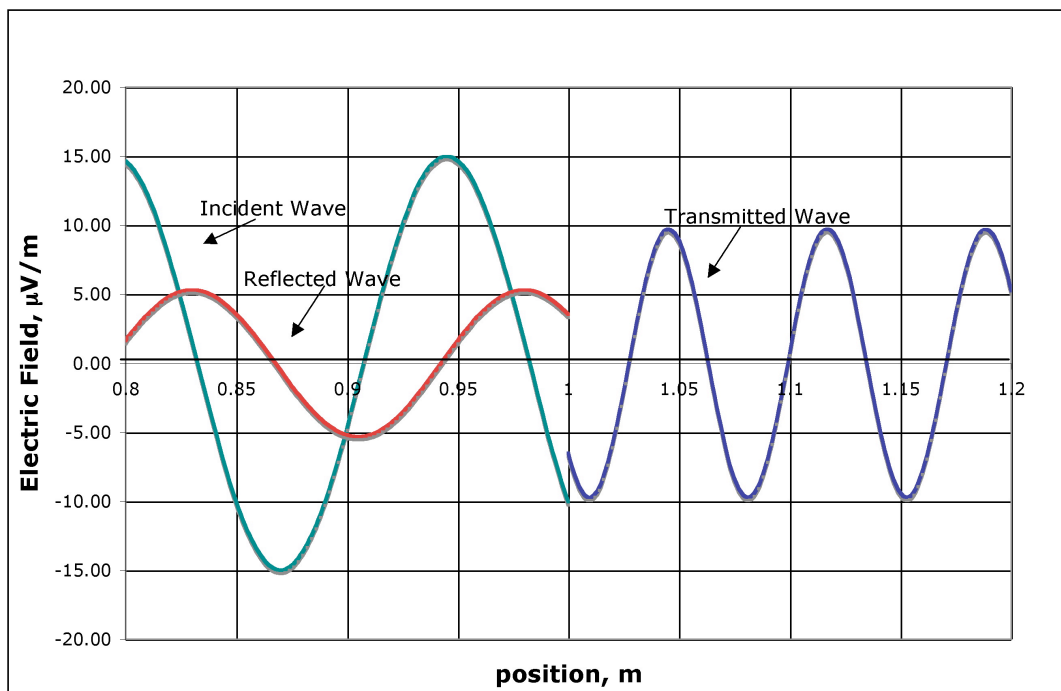
a) The array will be N antennas wide (treat each antenna as a point source) with a spacing of 1.3 m. If the width of the collector is 5000 m, what is the smallest value of N needed to have the central ($m=0$) diffraction beam match the width of the detector?

b) The maximum intensity that can be safely handled at any point on the collector is 500 W/m^2 . What is the maximum value of the intensity per antenna I_0 that can be safely transmitted?

4. An electromagnetic plane wave is traveling in the +z direction. The wave is polarized in the +x direction, and the amplitude of the electric field is 0.15 mV/m. The wave is traveling in vacuum. $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/(\text{Nm}^2)$ $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$
- a) What are the amplitude and direction of the magnetic field at an instant when the electric field is positive?
- b) The wave is normally incident on an interface to a dielectric material of $Z=180\Omega$. What is the refractive index of the dielectric material? What fraction of the intensity of the incident electromagnetic wave is transmitted through the interface? What is the amplitude of the transmitted electric field?

- c) The graph shows the incident, transmitted and reflected waves at one point in time, near the interface, which is located at position $z = 1.00$ m. What are the boundary conditions (2 equations) governing the behavior of the electric field at the interface? Draw a careful illustration of the physical meaning of your equations on the graph. The numbers in the table should be helpful in making a clear drawing.

z (m)	E_{inc} ($\mu\text{V/m}$)	E_{ref} ($\mu\text{V/m}$)	
0.97	7.50	4.85	
0.98	1.57	5.31	
0.99	-4.63	4.85	
1	-10.0	3.55	



5. A wave on a tightly stretched string is observed to have the form:

$$y(x,t) = (0.05\text{m})\sin[2.5\text{m}^{-1}x]\cos[500\text{s}^{-1}t].$$

a) What type of wave is this, and what are its wavelength and frequency?

b) Write this wave as the sum of an incident and reflected wave, giving numerical values for all the parameters for each (let phase shifts be zero).

c) What is the kinetic energy, $K(x,t)$ of this string if the mass per unit length is 0.030kg/m ?

6. The current flowing through an RLC circuit driven by an alternating voltage source $V_{ex}(t) = V_o \cos(\omega t)$ is:

$$I(t) = \frac{V_o \cos(\omega t - \delta)}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \text{ where } \tan \delta = \frac{\omega L - 1/\omega C}{R}$$

- a) For $L = 1.34 \times 10^{-4} \text{ H}$, $C = 22.3 \times 10^{-12} \text{ F}$, find the resonance frequency f_{res} of the circuit.
- b) If R is $5.73 \times 10^{-3} \Omega$, and $V_o = 4.0 \text{ V}$ find the average power dissipated in the resistor at the resonance frequency. Find the equations for the frequency ($f > f_{\text{res}}$) for which the power dissipated is 1/2 of the maximum value and 1/100 of the maximum value.

c) Now consider the case for f not equal to f_{res} . Find the value of the phase shift in the limit as R goes to zero. Insert this into the equation for the current, and find the equation for the current when R is zero. For this driven LC circuit, find the maximum value of the power in the circuit.