

Closed-book examination. One pre-prepared 8 1/2" x 11" sheet of notes may be used. Calculator may be used. Show work for partial credit. Include UNITS on all answers.

EXAM ADVICE: If you don't immediately see what to do on one problem, go on to the next problem and come back to the unfinished problems later.

**33 pts** 1. Acoustic waves are created in a liquid (mercury) of bulk modulus  $2.7 \times 10^{10} \text{ Pa}$  (bulk modulus =  $-\Delta P / (\Delta V / V)$ ), and volume density  $1.36 \times 10^4 \text{ kg/m}^3$ . The displacement amplitude is  $2.0 \times 10^{-5} \text{ m}$  and the frequency is  $1200 \text{ Hz}$ . The waves are initially traveling in the  $+x$  direction.

a) Find the velocity, wavelength and pressure amplitude  $\Delta P_0$  (assume the cross sectional area is constant so that  $\Delta V / V = \partial \xi / \partial x$ .)

b) If the waves are trapped between two rigid walls, what distances between the walls will allow standing waves to form? Sketch the three lowest frequency standing waves.

c) Write the wave equations for i) the standing displacement wave  $\xi(x,t)$ , and ii) the standing pressure wave  $\Delta P(x,t)$ . Assume the walls are at positions  $x = 0$  and  $x = L > 0$ .

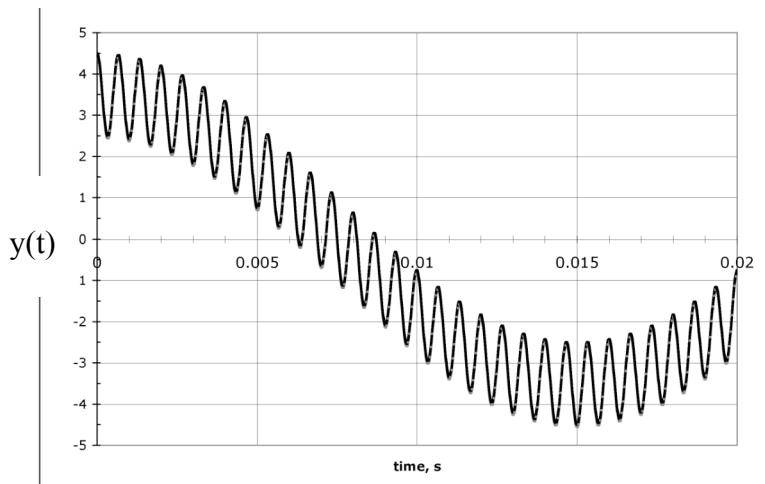
d) Write the equation for the kinetic energy per unit volume  $KE(x,t)/V$  in the standing wave. For the standing wave of second-lowest frequency (see part b), draw a graph of relative displacement ( $\xi/\xi_{\max}$ ) and kinetic energy ( $KE/KE_{\max}$ ) vs.  $x$  at a time when the displacement is at  $\sqrt{3}/2$  times its maximum value. Be clear.

**3 pts EXTRA CREDIT** (do this after all other problems!) How is the potential energy in a standing wave related to the kinetic energy? Explain how this compares to the situation for a traveling wave.

**(33 pts) 2.** Carefully draw the Fourier spectrum for each of the three waves indicated below. Label each peak with its frequency and magnitude.

a)  $V(t) = [3.4V + 0.85V\cos(2\pi \cdot 10^8 s^{-1}t)] \cdot \sin(2\pi \cdot 2.5 \times 10^7 s^{-1}t)$

b) The wave  $y(t)$  shown.



c)  $P(t) = P_o \operatorname{Re} \left[ [0.9e^{i\theta}] + [0.9e^{i\theta}]^2 + [0.9e^{i\theta}]^3 \right]$  where  $\theta = 2\pi(50\text{Hz})t$ .

**3 pts EXTRA CREDIT** (do this after all other problems!) the ratio of frequencies for two notes separated by an octave is  $f_8/f_1 = 2.0$ . Based on mathematical reasoning, what are the ratios of frequencies for a major third,  $f_3/f_1$  and a major fifth  $f_5/f_1$ ? (the mathematical values are close to the values actually used musically)

**(34 pts) 3.** A coaxial cable has inner radius  $a = 0.1$  mm and outer radius  $b = 3$ mm, and the gap between the conductors is filled with a glassy material with permittivity  $\epsilon = 5.6\epsilon_0$  for  $x < 0$ . At  $x = 0$ , the glassy material abruptly ends, leaving an air gap ( $\epsilon = \epsilon_0$ ) for  $x > 0$ .

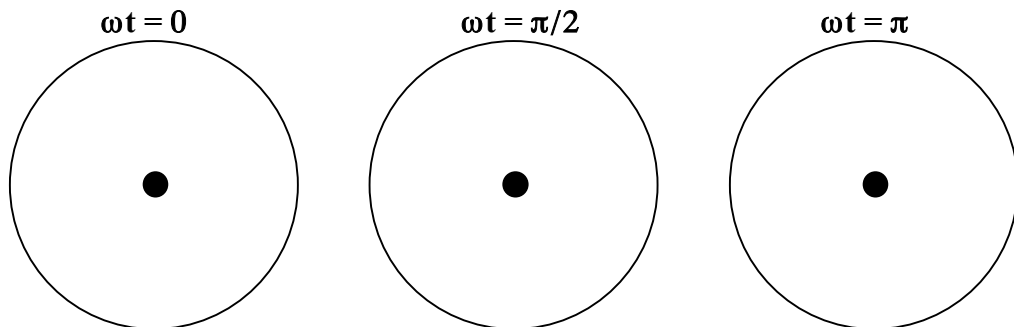
$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/(\text{Nm}^2) \quad \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

a) Find the impedance and electromagnetic wave speed of the cable for  $x < 0$  and for  $x > 0$ .

b) If an ac voltage  $V_0 \cos(\omega t)$  is applied to the cable at  $x \ll -0$ , what are the relative amplitudes of the transmitted waves for voltage and for current? (this means give the ratios  $V_t/V_i$  and  $I_t/I_i$ ).

- c) Find the power  $P(x,t)$  carried by the wave for  $x < 0$  assuming  $V_o = 4V$ . How is this related to the energy stored in the inductance and capacitance of the transmission line?

- d) Indicate the directions of the electric field and magnetic lines on the diagrams below for the times given. These show the cross section of the line with the  $+x$  direction pointing out of the page. Assume that  $E$  varies as  $\cos(kx - \omega t)$  and that at the position  $(x)$  of the cross-section  $kx = 2n\pi$ .



**3 pts EXTRA CREDIT** (do this after all other problems!) We showed how to use a quarter wave configuration to improve transmission. Describe how you might similarly create a strongly reflecting interface.