

# Homework #8

due Tuesday, April 8

1. Hirose & Lonngren Chapter 6 #1
2. Hirose & Lonngren Chapter 6 #4
3. Hirose & Lonngren Chapter 6 #6 (d) Also, find the fraction of the wave energy reflected.
4. Hirose & Lonngren Chapter 6 #10
5. *The particle in a box.* We have talked in class about how many of the laws of quantum mechanics can be derived simply from the properties of waves. One classic problem in quantum mechanics, known as “the particle in a box” or “the particle in an infinite potential well” is completely analogous to a guitar string with the ends nailed to a fret, and we can apply what we learned about standing waves to this problem to show that energy is quantized with nothing more than a bit of 3rd grade math.

Pretend like you have a quantum mechanical particle (like an electron, for example, all that matters is that it has some wavelike properties), and the poor little particle is stuck in a one dimensional valley between two cliffs. The cliffs are a distance  $L$  apart, and infinitely high (it would take an infinite potential energy to climb them), and it therefore can't escape.

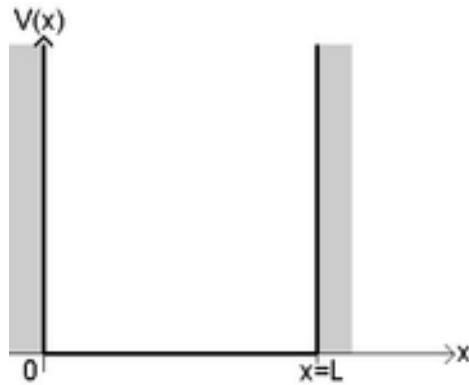


Figure 1: Illustration of an infinite potential well.

- (a) Draw the allowed wave patterns of the particle and write some quantization rules. *Hint: This is exactly the problem of a guitar string. The amplitude must be zero at the edge of the cliff, thus only certain wavelengths are allowed. What wavelengths are they?* Use  $n$  to denote an integer in your answer. For example, in Figure 6.3 in your book, one could describe the various harmonics with  $L = n\lambda/2$  where  $n = 1, 2, 3, \dots$
- (b) Next, the momentum of the particle is  $p = \hbar k$ . Here,  $\hbar$  is a constant, and  $k$  is our familiar wavenumber. We also know from freshman physics that the energy of a particle is given by  $E = \frac{1}{2}mv^2 = p^2/2m$ . What is the expression for the energy of the various energy levels?
- (c) How is the energy of the excited states of the particle related to that of the ground state? *In quantum-speak, the word “ground state” is used analogously to “fundamental”, and “excited state” corresponds to “harmonic”.* (Ans.  $E_n = n^2 E_1$ .)