The Doppler shift

1. In class, we derived a “general” expression for the Doppler shift which takes into account the motion of both the source and the observer. In fact, equation 8.7 assumes that the motion of the source and the observer occur along a straight line.

Consider the following, more general, situation. A car is traveling along a road at 50 miles per hour. A train is approaching the car with a constant speed of 25.0 m/s on a track that is running parallel to the road, but 30.0 m away from it. The train’s horn emits a frequency of 500 Hz.

\[ \text{50 mph} = 22 \text{ m/s} \]

a) Draw a diagram, similar to Figure 8.6 in your book, showing the velocity and position of the car, the train, and the propagation of the sound waves.

b) Write a more general equation for the Doppler shift that could be used in this situation in terms of the angle between the line of sight to the train and the direction of the road. (This is a generalization of Example 1 in your book.)

If both source and observer are moving along the same line \( v' = \frac{v_s - u_o}{c_s - u_o} v_o \). In this more general situation we must take the velocity components along the line of sight.

\[ v' = \frac{c_s - u_s \cos \theta}{c_s - u_s \cos \theta} v_o \]

\( \cos \theta = \frac{40}{(10)^2 + (30)^2} = 0.8 \]

\( u_s > 0 \quad u_o < 0 \)

\( \text{the car is in the direction of travel} \)

\( c_s \approx v_s \approx 340 \text{ m/s} \)

\( v' = \frac{340 \text{ m/s} + 22 \text{ m/s} (0.8)}{340 \text{ m/s} - 25 \text{ m/s} (0.8)} \quad 500 \text{ Hz} = 559 \text{ Hz} \)

c) What frequency do the passengers in the car hear at the instant when the train is 40 m from its point of closest approach?

\( 30 \text{ m} \quad \frac{22 \text{ m/s}}{3} \quad \frac{340 \text{ m/s} + 22 \text{ m/s} \times 0.8}{340 \text{ m/s} - 25 \text{ m/s} \times 0.8} \quad 500 \text{ Hz} = 559 \text{ Hz} \)

d) Qualitatively explain how the sound of the train’s horn evolves as it approaches and passes the car (as heard by the passengers inside the car).

Initially, \( v' > v_o \). At the point of closest approach, \( v' = v_o \). As the train recedes, \( v' < v_o \).
e) What is the minimum and maximum frequency heard by the observers in the car?

In the limit of very small $\theta$ (when the train is very far away, $\cos \theta \approx 1$), our more general equation reduces to Equation 8.7.

By our sign convention, as the train approaches $U_s > 0$, $U_o < 0$,
and as it recedes $U_s < 0$, $U_o > 0$.

Approaching: $\nu' = \frac{540+22}{340-25} \frac{m/s}{s} = 574 Hz$  $\nu' = \frac{(340+22)(500Hz)}{(340-25)(500Hz)} = 435 Hz$

f) How would you modify the equation found in b) for the more general situation where the car and the train are not moving in parallel directions?

\[
\nu = \frac{C_s - U_s \cos \theta_0}{C_s - U_s \cos \theta_0} \frac{v_0}{v_0} \]

take the components of velocity in direction of line of sight

2. The speed of light in water is $2.25 \times 10^8$ m/s (about $\frac{2}{3}$ the speed of light in a vacuum). A beam of high-speed electrons emits Cerenkov radiation in the water, the wavefront being a cone of angle $52^o$. $32^o$

a) For some time $t>0$, draw a diagram showing the position of the electrons, and wavefronts of the electromagnetic radiation emitted at time $t=0$. Carefully show the position of the angle $\theta$.

b) Find the speed of the electrons in water.

\[
\cos \theta = \frac{c}{n} \frac{1}{V_e} \quad V_e = \frac{c}{n} \frac{1}{\cos \theta} = 2.65 \times 10^8 \text{ m/s}
\]