Name:________________________

# multidimensional waves and waves in nonuniform media

Consider a square drum (a membrane stretched over a square frame with uniform tension, T). Let m be the number of half wavelengths in the x direction and n be the number of half wavelengths in the y direction.

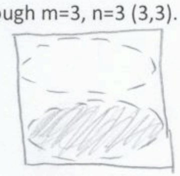
m=1
n=1

*This shows the fundamental, where the area enclosed by the dotted line shows an antinode.*

*This shows the m=2, n=1 mode, where at a snapshot in time the shaded and unshaded antinodes is have opposite displacements.*

Note that the nodes and the antinodes are now 2 dimensional. Continue drawing the vibrational modes up through m=3, n=3 (3,3).

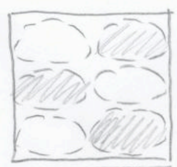
m=1
n=2



m=1
n=3

m=3
n=1

m=2
n=3



m=3
n=2

m=3
n=3

Which modes are degenerate (which in this case means that they have the same frequency)?

m=2 n=1
m=1 n=2

m=1 n=3
m=3 n=1

m=2 n=3
m=3 n=2

How does the displacement of the membrane evolve over 1 period for a particular vibrational mode?

An antinode that initially had a positive maximum displacement will have zero displacement 1/4 of a period later. It will have a maximum negative displacement 1/4 of a period after that. After 1 period, it returns to its initial position.

Would your results change if the tension in the x direction were different from the y direction?

Yes! In that case, you would have a different wave velocity for propogation in the x and y directions, and thus a different wave equation describing the motion in each direction. You wouldn't get these well defined nodes and antinodes.

In class, we discussed the evolution of a pulse as it propagates to the bottom of a vertical string. Can you explain in terms of the conservation of energy why the amplitude increases.

Energy is conserved, thus as the velocity decreases, and the pulse narrows, the amplitude must increase to conserve energy.

A spherical wave is radiating from a point source and is described by the following:

$$y(r,t) = \left(\frac{25.0}{r}\right)\sin(1.25r - 1870t)$$ where y is in Pascals, r in meters, and t in seconds.

What is the maximum pressure amplitude 4 m from the source?

$\frac{25}{4}$ Pascals $= 6.25\ N/m^2$

What is the speed of the wave?

$\frac{1870}{1.25}\ m/s = 1496\ m/s$ sound is propagating through water

Find the intensity of the wave in dB at a distance 4.00 m from the source.

This is a pressure wave, not a displacement wave, so it is in the form $\frac{F}{A}$ analogous to equation 5.22, so the amplitude is $Yk\mathcal{E}_0$, (here we use $M_B$ instead of Y since it is water)

$\mathcal{E}_0 = \frac{25.0}{rM_Bk} = \frac{25\ N/m^2}{4(2.1\times10^9 N/m^2)(1.25m^{-1})} = 2.4\times10^{-9}m$ $I = \frac{1}{2}\rho_v c_w \omega^2 \mathcal{E}_0^2 = \frac{1}{2}(1496)(10^3 kg/m^3)(1870)^2(2.4\times10^{-9})^2 = 1.48\times10^{-5} W/m^2$

$dB = 10\log_{10}\left(\frac{I}{1.0\times10^{-12} W/m^2}\right) =$ 7.17 dB

Find the instantaneous pressure at 5.00m from the source at 0.0800s.

$y(5m, 0.08s) = \left(\frac{25}{5}\text{ Pascals}\right)\sin(\underbrace{1.25(5) - 1870(0.0800)}_{\text{in radians}})$

$= 4.6\ N/m^2$