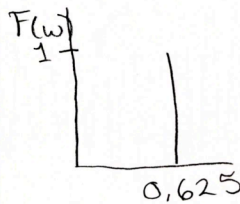
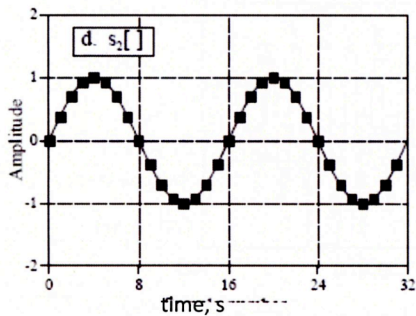


Name: _____

Fourier analysis

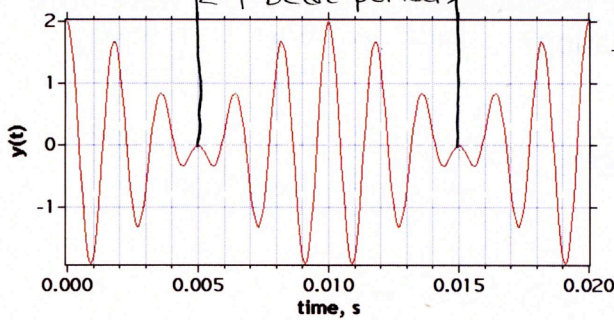
State Fourier's theorem in your own words.
 any waveform can be written as the sum of periodic waves

Graph the fourier spectrum of each wave, and write the Fourier series. This is a single sine wave with period 16s and amplitude 1. $\Rightarrow \nu = \frac{1}{16s}$ $\omega = \frac{2\pi}{16s} = \frac{\pi}{8} s^{-1}$



$$f(t) = \sin\left(\frac{\pi}{8}t\right)$$

beat pattern



$$T_{\text{beat}} = 0.010s \quad \nu_{\text{beat}} = \frac{1}{0.01s} = 100\text{Hz}$$

there are about 5.5 phase cycles in 1 beat

$$T_{\text{phase}} = \frac{0.01s}{6} = 0.00166$$

$$\nu_{\text{phase}} = 600$$

$$\nu_1 - \nu_2 = 100\text{Hz}$$

$$\frac{\nu_1 + \nu_2}{2} = 600 \Rightarrow \nu_1 + \nu_2 = 1200\text{Hz}$$

$$\nu_1 + \nu_2 = 1200\text{Hz}$$

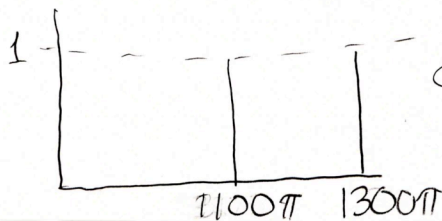
$$\nu_1 - \nu_2 = 100\text{Hz}$$

$$\frac{2\nu_1}{2} = 1100\text{Hz} \quad \nu_1 = 550$$

$$\nu_2 = 1200 - 550 = 650$$

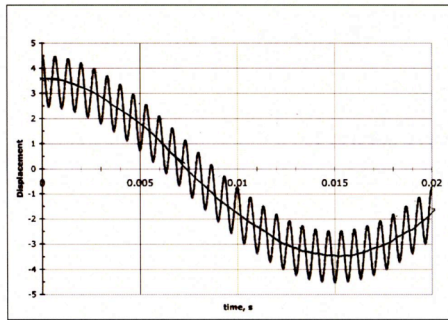
$$\Rightarrow \omega_2 = 1300\pi \text{ rad/s}$$

$$\omega_1 = 1100\pi \text{ rad/s}$$



$$f(t) = \cos(1100\pi t) + \cos(1300\pi t)$$

low frequency wave: $A_1 = 3.5 \text{ cm}$ $T_1 = 0.03$ $\nu = 33 \text{ Hz}$

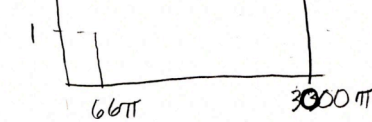


high frequency wave: $A_2 = 1 \text{ cm}$

$$T_2 = \frac{0.02}{30} = 6.6 \times 10^{-4} \text{ s} \quad \nu_2 = 1500 \text{ Hz}$$

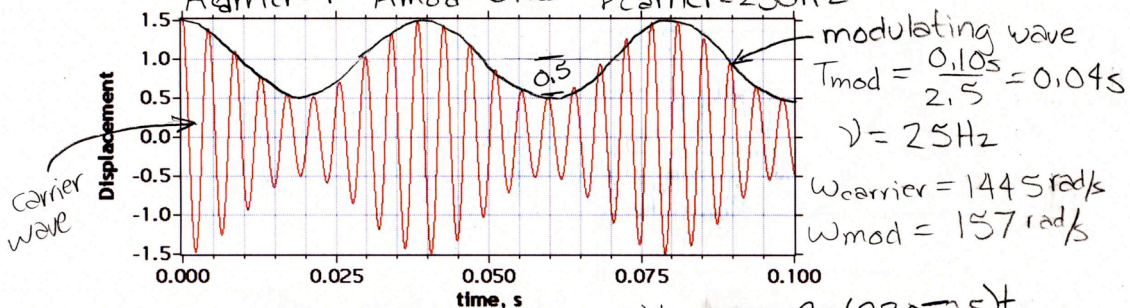
$$\omega_1 = 2\pi \cdot 33 = 66\pi$$

$$\omega_2 = 2\pi \cdot 1500 = 3000\pi$$



$$f(x) = 3.5 \sin 66\pi t + 1 \sin 3000\pi t$$

this is amplitude modulation $T_{\text{carrier}} = \frac{0.100}{23} = 0.00434 \text{ s}$
 $A_{\text{carrier}} = 1$ $A_{\text{mod}} = 0.25$ $\nu_{\text{carrier}} = 230 \text{ Hz}$



$$f(x) = \cos 2\pi(230)t + 0.25 \cos 2\pi(230+25)t + 0.25 \cos 2\pi(230-25)t$$

How long would you have to sample each of the above waveforms to get most of the information?

$$\text{pulse width} \times \text{bandwidth} = 2\pi$$

$$\text{pulse width} = 2\pi / \text{bandwidth}$$

for the beat waveform:

$$T = \frac{2\pi}{\Delta\omega} = \frac{1}{\Delta\nu} = \frac{1}{100 \text{ Hz}} = 0.01 \text{ Hz} \text{ which is equal to one beat waveform}$$

for am wave

$$T = \frac{1}{255 - 205 \text{ s}^{-1}} = \frac{1}{50 \text{ s}^{-1}} = 0.02 \text{ s}$$