Recitation 5

Waves and energy transport

Name:_____________________________

A transverse sinusoidal wave is generated at one end of a long, horizontal string by a bar that moves up and down through a distance of 1.0 cm. The motion is continuous and is repeated 120 times per second. The string has linear density 120 g/m and is kept under a tension of 90 N

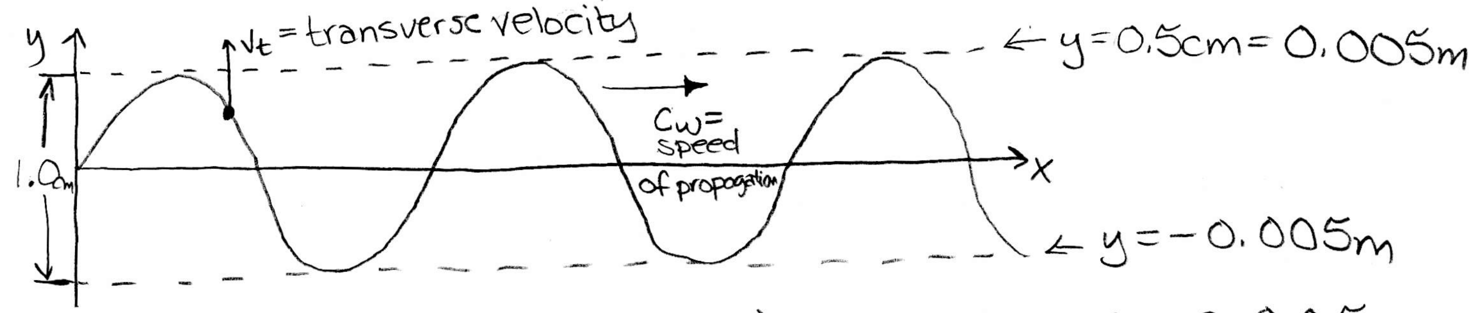
a) Find the maximum value of the transverse speed.

b) The maximum value of the transverse component of the tension. *Hint: draw the string and draw the direction of the tension for various points on the string.*

c) Show that the maximum value occur at the same phase values for the wave. What is the transverse displacement at these phases?

d) At what point is the power transfer maximum? *(Hint: Power is the rate of doing work. What is the definition of work?)*

Let's first draw a picture of our wave



$y(x,t) = A\sin(kx-\omega t)$ where $A = 0.005\,m$

where $\omega = 2\pi f = 2\pi(120\,s^{-1}) = 754\,Hz$

Now find k: $k = 2\pi/\lambda$ $C_w = \lambda f \Rightarrow \lambda = C_w/f$

$$C_w = \sqrt{\frac{T}{\rho_e}} = \sqrt{\frac{90N}{0.120\,kg/m}} = 27\,m/s$$

$$\lambda = 27\,m/s \,/\, 120\,s^{-1} = 0.225\,m \quad k = \frac{2\pi}{0.225\,m} = 27.9\,m^{-1}$$

$$y(x,t) = 0.05\,m \sin(27.9x - 754t)$$

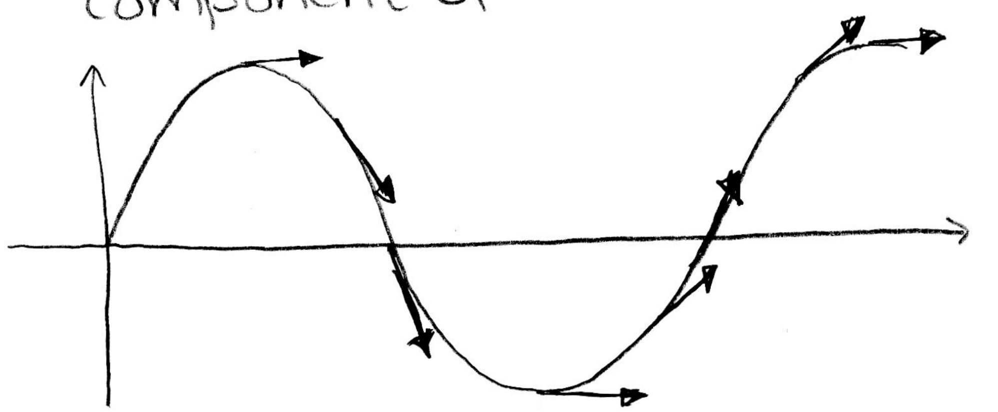
the transverse velocity is the velocity at which a fixed point on the string moves up and down.

$$V_t = \frac{dy}{dt} = -\omega A\cos(kx-\omega t)$$

$|V_t|$ will be a maximum when $\cos(kx-\omega t) = \pm 1$

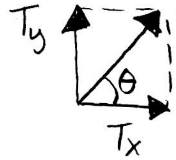
$$|V_t|_{max} = \omega A = (754\,s^{-1})(0.005\,m) = 3.77\,m/s$$

Now find the maximum of the transverse component of the tension



The tension is directed tangent to the string at each spot.

The transverse component of tension will be maximum when the slope is maximum.



Tension resolved into components

The slope is $\frac{dy}{dx}$. Also $\frac{\Delta y}{\Delta x} = \tan\theta$.

$T_x = T\cos\theta$

$T_y = T\sin\theta$

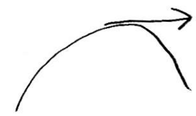
$$\frac{dy}{dx} = kA\cos(kx - \omega t)$$

$$\left.\frac{dy}{dx}\right|_{max} = (27.9\,m^{-1})(0.05\,m) = 0.14 = \tan\theta$$

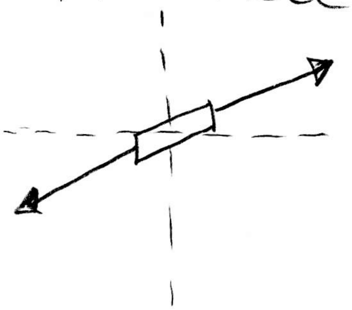
$$\theta = \tan^{-1}(0.14) = 7.9°$$

$$T_y = T\sin\theta = 90\,N(\sin 7.9°) = 12.4\,N$$

Note that at the top of the wave, the transverse component of the tension is zero.



Also note that at the point of maximum tension, the string is being maximatlly stretched! This is the point where the potential energy of the string is greatest.



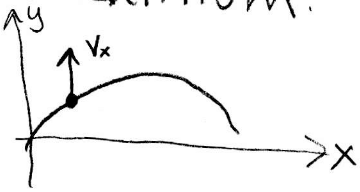
Note that both the tension and the transverse velocity were a maximum when $\cos(kx - \omega t) = 1$

Thus the kinetic and potential energies are a maximum at the same time! (unlike a block on a spring)

See pg. 63 of your book.

At what point is the power transfer maximum?

$Work = \vec{F} \cdot \vec{d}$ where d is the displacement a point on our string is displaced in the transverse direction. The force is provided by the tension, T.



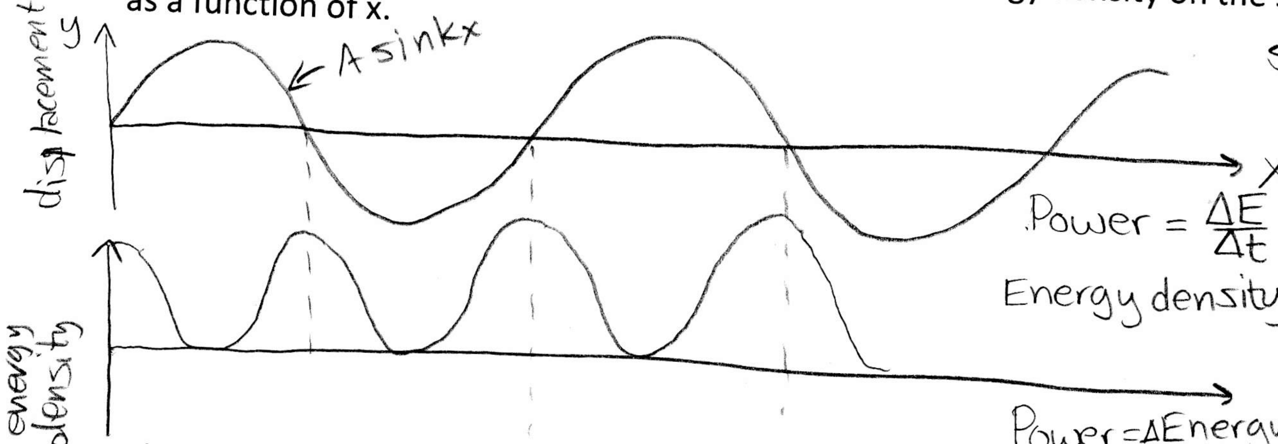
Power is the rate of doing work.

$$P = \frac{dW}{dt} = T_y \frac{dy}{dt} = T \sin\theta \frac{dy}{dt}$$

because we took a dot product we are left with the component of tension parallel to the velocity

So we see from the results of the previous questions that the maximum power along the string is in the places where $y=0$ and $\frac{dy}{dt}$ and $\frac{dy}{dx}$ are maximum.

Carefully draw the displacement of the string and the energy density on the string as a function of x.



snapshot in time

$\text{Power} = \frac{\Delta E}{\Delta t}$

$\text{Energy density} = \frac{\Delta\text{Energy}}{\Delta\text{length}}$

$\text{Power} = \Delta\text{Energy density} \times \frac{\Delta\text{length}}{\Delta\text{time}}$

What is the maximum power transferred along the string?

$\text{Energy density} = \frac{\text{Power}}{\text{propagation velocity}} = \text{Power}/c_w$

$P = \rho_\ell c_w \omega^2 A^2 \cos^2\theta$ (from your book or notes)

max when $\cos^2\theta = 1$ $= \rho_\ell c_w \omega^2 A^2 = (0.120\,\text{kg/m})(27\,\text{m/s})(754\,\text{s}^{-1})^2 \times (0.005\,\text{m})^2$

$= 46.04\,\text{W}$

If you want to get this from part d) note that for small angles $\sin\theta \approx \tan\theta = \frac{dy}{dx}$

$P = T\sin\theta \frac{dy}{dt} = T\frac{dy}{dx}\frac{dy}{dt} = T(kA\cos(kx-\omega t))(\omega A\cos(kx-\omega t))$

What is the minimum power transfer along the string? What is the transverse displacement for conditions under which this minimum power transfer occurs?

The minimum power is at the "top" of the wave, there the energy transfer is zero!

For a string segment, δx, what is applying the force that is causing its acceleration?

For any segment δx, it is the adjacent segments that are exerting force on it.

Note that the segment at the "top" of the wave (the crest) isn't moving!!

$\therefore P = \vec{F}\cdot\vec{v} = 0$