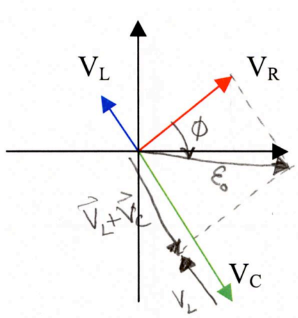
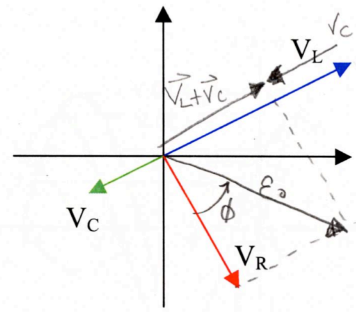
Name: ___________________________

Forced Oscillations

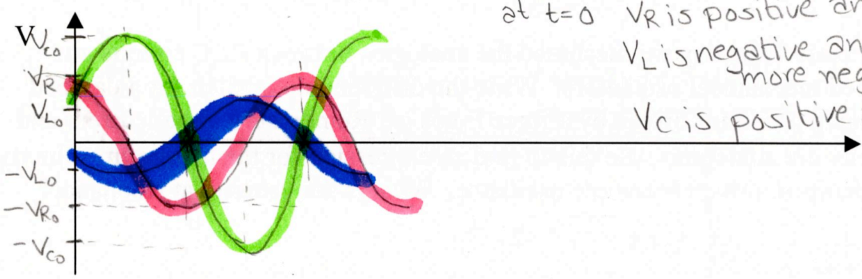
1. Using the following phasor diagrams representing series RLC circuits, draw the vector representing the driving voltage and estimate the phase shift between the current and the driving voltage.



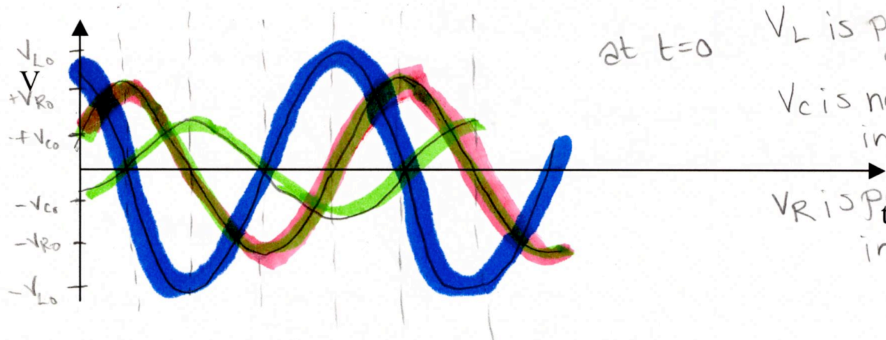
$\tan\phi = \frac{|\vec{V}_L+\vec{V}_c|}{|\vec{V}_R|}$



2. Assuming the snapshots above were taken at t=0, for each circuit, draw the voltage across each component as a function of time.



at $t=0$ V_R is positive and decreasing
V_L is negative and becoming more negative
V_c is positive and increasing

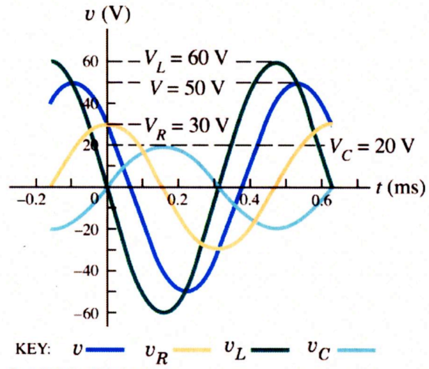


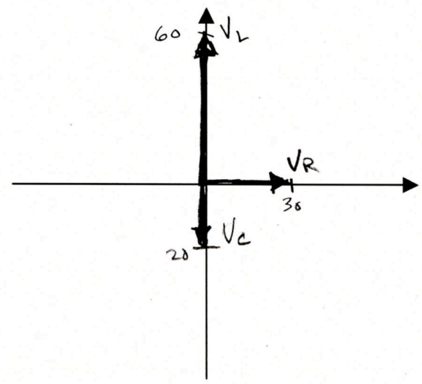
at $t=0$ V_L is positive and decreasing
V_c is negative and increasing
V_R is positive and increasing

3. Qualitatively explain why there is a phase difference between the components in the circuit.

V_R is proportional to I
V_L is proportional to di/dt and is maximum when $i=0$
V_c is 180 out of phase with V_L. It also has a maximum absolute value when $i=0$

3. Draw a phasor diagram representing the following circuit at time t=0.





4. In class Thursday we discussed the analogies between RLC circuits and damped mechanical oscillators. Write the differential equation for a damped harmonic oscillator driven by a force $F_o\cos \omega t$. Remembering that current and velocity are analogous, use this to find an expression for the maximum velocity of a damped, driven harmonic oscillator. What is the velocity at resonance?

$$m\frac{d^2x}{dt^2} + f\frac{dx}{dt} + kx = F_0 \cos\omega t$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = \mathcal{E}_0 \cos\omega t$$

$$i = \frac{\mathcal{E}_0}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$\mathcal{E}_0 \to F_0$

$\frac{1}{C} \to k$

$R \to f$

$L \to m$

$\frac{dq}{dt} \to \frac{dx}{dt}$

$$\frac{dx}{dt} = \frac{F_0}{\sqrt{f^2+\left(\omega m - \frac{k}{\omega}\right)^2}}$$

at resonance $\omega = \omega_0 = \sqrt{\frac{k}{m}}$

$$\left.\frac{dx}{dt}\right|_{\omega=\omega_0} = \frac{F_0}{\sqrt{f^2+\left(m\sqrt{\frac{k}{m}} - k\sqrt{\frac{m}{k}}\right)^2}}$$

$$= \frac{F_0}{\sqrt{f^2+\left(\sqrt{mk} - \sqrt{mk}\right)^2}}$$

$$= \frac{F_0}{f}$$