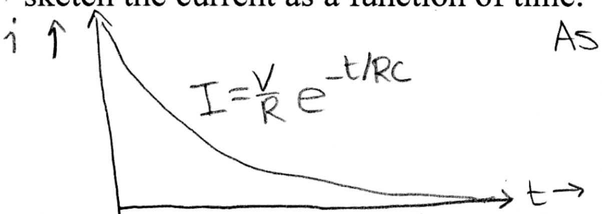
Review of AC circuits

Name:____________________

Suppose you connect a battery to an uncharged capacitor. Describe or qualitatively sketch the current as a function of time.



As $t \to \infty$ $i \to 0$ current stops flowing
As $t \to \infty$ this behaves like an open circuit (as if the wire were cut.)

Now suppose you connect the capacitor to a very low frequency AC generator. What happens to the current as a function of time?

The capacitor will alternately charge and discharge if the period is longer than the time constant of the circuit.

Suppose that you gradually increase the frequency of the ac generator. How does the behavior of the circuit change?

If you increase the frequency the capacitor has less time to charge and discharge. If the period is short compared to the time constant of the circuit, the capacitor never becomes charged enough to significantly impede the flow of current. It behaves like a "short circuit".

Now consider a circuit with only an inductor in it. Under what circumstances does an inductor produce an emf? $V = L\frac{di}{dt}$ (when the current is changing)

How does the current in the inductor circuit behave when you hook it up to a battery?

A battery provides constant current so $di/dt = 0$ Thus the inductor does not provide back emf to impede the flow of current. It acts like a short circuit.

How does this behavior change if you hook replace the battery with an ac generator? How does this change as the generator frequency is increased?

With an ac generator, the current is constantly changing and di/dt is no longer zero. At high frequencies there is enough back emf to significantly impede the flow of current.

Compare and contrast the role of the capacitor and the inductor in impeding the flow of current with that of a resistor. The amount by which the capacitor and inductor allow or impede the flow of current is frequency dependent.

We quantify the "effective resistance" to the flow of current in the circuit in terms of the reactances, given by:

$$X_c = \frac{1}{\omega C} \qquad X_L = \omega L$$

What is the reactance of a 1 mH inductor at a) 60 Hz, b) 600 Hz, and c) 6kHz? Do your answers agree with the intuition you developed on the previous page?

$X_L = \omega L$ where $\omega = 2\pi f$

a) $X_L = 2\pi(60\,Hz)(.001\,H) = 0.377\Omega$

b) @ 600 Hz $X_L = 3.77\Omega$

c) @ 6000 Hz $X_L = 37.7\Omega$

reactance increases as frequency increases

What is the reactance of a 1 nF capacitor at a) 60 Hz, b) 600 Hz, and c) 6kHz? Do your answers agree with the intuition you developed on the previous page?

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ a) @ 60 Hz $X_c = \frac{1}{2\pi(60)(1\times10^{-9}F)} = 2.7\times10^{6}\Omega$

b) @ 600 Hz $= 2.7\times10^{5}\Omega$

c) @ 6000 Hz $= 2.7\times10^{4}\Omega$

Now suppose you connect an inductor, a capacitor, and the resistor in series with an ac generator. If the current in the circuit is $i = I\cos\omega t$ where I is the peak current, will all of the component reach their peak voltage at the same time? If not, describe.

No. The voltage on the capacitor is the maximum, it maximally impedes the flow of current.
The voltage on the inductor is maximum when the rate of change of the current is maximum.
The voltage on the resistor is proportional to the current

Draw as many analogies as you can between the RLC circuit and the damped mechanical oscillator discussed in class on Thursday.

both shuttle energy back and forth between two forms (potential and kinetic for the spring system, and electric and magnetic fields for the RLC circuit)
the resistor and friction are analogous in that they dissipate energy from the system.
both will resonate at a characteristic frequency.