ch 7, # 1,

(a) $\frac{5W}{\pi \cdot (10m)^2} = 0.13 W/m^2$

(b) $\frac{5W}{\pi \cdot r^2} = 10^{-6} W/m^2 \quad \therefore r = 3.6 km$

#1 Ch7 #3 $P = 50kW$

a) $P = IA$ ← area   at $r = 1$ mile $= 1610m$

power ↑ ↑ intensity $A = 4\pi r^2 = 4\pi(1610m)^2$

$$I = \frac{P}{A} = \frac{50kW}{4\pi(1610m)^2} = 1.5\ mW/m^2$$

b) for a wave travelling spherically outward

use 7.6 $\mathcal{E}_0(r) \sim \frac{1}{r}$ ← radial distance

↑ amplitude

$$\frac{\mathcal{E}(1mile)}{\mathcal{E}(10mile)} = \frac{(1/1mile)}{(1/10miles)} = \frac{10}{1}$$

c) The intensity is $3\mu W/m^2 = 3\times10^{-6} W/m^2$ at a radial distance of:

$$P = IA = I(4\pi r^2) \quad \therefore r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{50\,000W}{4\pi(3\times10^{-6}W/m^2)}}$$

$$= 36km = 22.6 \text{ miles}$$

#2 Ch7 #6

$$\mathcal{E}(r,t) = (A/r)\sin(kr - \omega t)$$

$$\frac{\partial\mathcal{E}}{\partial r} = \frac{-A}{r^2}\sin(kr-\omega t) + \frac{kA}{r}\cos(kr-\omega t)$$

$$\frac{\partial^2\mathcal{E}}{\partial r^2} = \frac{2A}{r^3}\sin(kr-\omega t) - \frac{kA}{r^2}\cos(kr-\omega t)$$

$$-\frac{kA}{r^2}\cos(kr-\omega t) - \frac{k^2A}{r}\sin(kr-\omega t)$$

$$= \left(\frac{2A}{r^3} - \frac{k^2A}{r}\right)\sin(kr-\omega t) - \frac{2kA}{r^2}\cos(kr-\omega t)$$

$$\frac{\partial \mathcal{E}}{\partial t} = \frac{-\omega A}{r} \cos(kr-\omega t)$$

$$\frac{\partial^2 \mathcal{E}}{\partial t^2} = -\frac{\omega^2 A}{r} \sin(kr-\omega t)$$

Plugging this into Eqn. 7.8 andusing $c = \frac{\omega}{k}$

$$\frac{\partial^2 \mathcal{E}}{\partial t^2} = c^2\left(\frac{\partial^2 \mathcal{E}}{\partial r^2} + \frac{2}{r}\frac{\partial \mathcal{E}}{\partial r}\right)$$

$$-\frac{\omega^2 A}{r} \sin(kr-\omega t) =$$

$$\frac{\omega^2}{k^2}\Bigg[\cancel{\frac{2A}{r^3}\sin(kr-\omega t)} - \frac{k^2 A}{r}\sin(kr-\omega t)$$

$$\cancel{-\frac{2kA}{r^2}\cos(kr-\omega t)}$$

$$+\cancel{\frac{2}{r}\left(\frac{-A}{r^2}\sin(kr-\omega t)\right.} + \cancel{\left.\frac{kA}{r}\cos(kr-\omega t)\right)}\Bigg]$$

$$-\frac{\omega^2 A}{r}\cancel{\sin(kr-\omega t)} = \frac{\omega^2}{\cancel{k^2}}\left[-\frac{\cancel{k^2}A}{r}\cancel{\sin(kr-\omega t)}\right]$$

$$-\frac{\omega^2 A}{r} = -\frac{\omega^2 A}{r}$$

#3 Ch7 #11 $T_s = 5 N/m$ $\rho_s = 40 g/m^2 = 0.040 kg/m^2$

$a = 30cm = 0.30m$ $b = 20cm = 0.20m$

a) $C_w = \sqrt{\frac{T_s}{\rho_s}} = \sqrt{\frac{5 N/m}{.040 kg/m^2}} = 11.2 m/s$

b) using 7.19 with m=1 n=1 (lowest order)

$\omega = C_w \pi \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 11.2 m/s \, \pi \sqrt{\frac{1}{(0.3m)^2} + \frac{1}{(0.2m)^2}} = 211 Hz$

$f_{11} = \frac{\omega}{2\pi} = 33.7 Hz$

c) higher order

$\omega(m=2, n=1) = C_w \pi \sqrt{\frac{(2)^2}{(0.20)^2} + \frac{1}{(0.30)^2}} = 371 Hz$ $f_{21} = 59 Hz$

$\omega(m=1, n=2) = C_w \pi \sqrt{\frac{1}{(0.20)^2} + \frac{(2)^2}{(0.30)^2}} = 293 Hz$ $f_{12} = 46.7 Hz$

$\omega(m=2, n=2) = C_w \pi \sqrt{\frac{(2)^2}{(0.20)^2} + \frac{(2)^2}{(0.30)^2}} = 2 C_w \pi \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 422 Hz$

$f_{22} = 67 Hz = 2 f_{11}$

note that generally higher order frequencies are <u>not</u> integer multiples of the lowest order frequency

in the special case of m=n, however, the frequency <u>is</u> an integer multiple of the lowest order frequency

6-8

for. $\xi(x,y,z,t)|_{x=0} = \xi(x,y,z,t)|_{x=a} = \xi(x,y,z,t)|_{y=0} = \xi(x,y,z,t)|_{y=b}$

$= \xi(x,y,z,t)|_{z=0} = \xi(x,y,z,t)|_{z=c} = 0$.

$\Rightarrow \xi = \xi_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \sin\frac{l\pi z}{c} \sin\omega t$

$$\frac{\partial^2\xi}{\partial x^2} = -\left(\frac{m\pi}{a}\right)^2 \xi_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi x}{b} \sin\frac{l\pi z}{c} \sin\omega t$$

$$\frac{\partial^2\xi}{\partial y^2} = -\left(\frac{m\pi}{b}\right)^2 \xi_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi x}{b} \sin\frac{l\pi z}{c} \sin\omega t$$

$$\frac{\partial^2\xi}{\partial y^2} = -\left(\frac{l\pi}{c}\right)^2 \xi_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi x}{b} \sin\frac{l\pi z}{c} \sin\omega t.$$

$$\frac{\partial^2\xi}{\partial t^2} = -(\omega)^2 \xi_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi x}{b} \sin\frac{l\pi z}{c} \sin\omega t$$

plug into : $\frac{\partial^2\xi}{\partial t^2} = c_s^2\left(\frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial y^2} + \frac{\partial^2\xi}{\partial z^2}\right)$

$$\omega^2 = c_s^2\left\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2\right\}$$

$$\omega_{mnl} = \pi c_s\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{c}\right)^2}$$