Physics 273

Homework # 1

Solutions

• H & L. Chapter 1, # 4)

• We have $\omega = \sqrt{K/M}$, want to show that $M \frac{d^2x}{dt^2} = -Kx$

$\Rightarrow$ (a) $x = A \sin \omega t$

$$\Rightarrow \frac{d^2x}{dt^2} = -A\omega^2 \sin\omega t \ , \quad \omega^2 = K/M$$

$$\Rightarrow \frac{d^2x}{dt^2} = -A\, K/M \sin\omega t \Rightarrow M\frac{d^2x}{dt^2} = -\cancel{M} \cdot A \cdot K/\cancel{M} \sin\omega t$$

$$= -K \overbrace{A \sin\omega t}^{x} = -Kx \ \blacktriangle$$

(b) $x = A \sin\omega t + B \cos\omega t$

$$\Rightarrow \frac{d^2x}{dt^2} = -A\omega^2 \sin\omega t - B\omega^2 \cos\omega t$$

$$\Rightarrow M\frac{d^2x}{dt^2} = \cancel{M} \cdot \left(-A \cdot K/\cancel{M} \sin\omega t - B \cdot K/\cancel{M} \cos\omega t\right)$$

$$= -K\left(A\sin\omega t + B\cos\omega t\right) = -Kx \ \blacktriangle$$

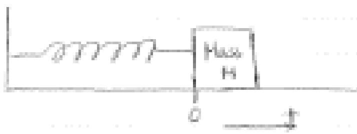
(c) $x = A\cos(\omega t + \phi)$

$$\Rightarrow \frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \phi) = -A \cdot K/M \cos(\omega t + \phi)$$

$$\Rightarrow M\frac{d^2x}{dt^2} = \cancel{M}\left(-A \cdot K/\cancel{M} \cos(\omega t + \phi)\right) = -K \overbrace{A\cos(\omega t + \phi)}^{x} = -Kx \ \blacktriangle$$

. H&L, Chapter 1, #5)

$m = 1.5\ kg$, $\Delta x = 10\ cm$ to the left , 20 Oscillations/min



a) $F = ma = m\frac{d^2x}{dt^2} = -K\Delta x$

$a = \frac{d^2x}{dt^2}$ where $x = A\cos\omega t \Rightarrow a = -A\omega^2\cos\omega t$, $A = \Delta x$

$\Rightarrow \cancel{-}K\cancel{\Delta x} = \cancel{-}\overset{\Delta x}{\cancel{A}}\omega^2\cos\omega t \cdot m$

$\Rightarrow K = m\omega^2\cos\omega t$

$= (1.5)(2.1)^2\cos(2.1(60/20)) = 6.57 \approx \underline{6.6\ N/m}$

another way to do part (a):

$\nu = 20$ Cycles/min $\Rightarrow$ 1/3 Cycle/sec $\Rightarrow$

we have $\nu = 1/4\pi^2\sqrt{K/M}$ (by Equation 13 in H&L)

$\Rightarrow \nu^2 = 1/4\pi^2 \cdot K/M \Rightarrow K = \frac{2\pi^2}{3} = 6.57 \approx \underline{6.6\ N/m}$

(b) $x = A\cos\omega t$;

$\omega = \sqrt{K/M} = \sqrt{\frac{6.57}{1.5}} = 2.09 \approx 2.1$

$\Rightarrow x = -10\,(cm).\underline{\cos(2.1t)}$

(c)

Total Energy associated with the Oscillation:

$T.E. = 1/2\,KA^2 = 1/2\,(6.6)(.1m)^2 = \underline{3.3\times10^{-2}\ J}$