Name: ____________________

I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

Signature: ____________________

Make sure your exam has five pages. Continue on the back of the page if you run out of space. You may use two 4×6 recipe cards with equations written on it. You may also use a calculator. Show all of your work.

Physics 273 Exam 2

1. Three successive resonance frequencies in an organ pipe are 1310 Hz, 1834 Hz, and 2358 Hz at 20° C and 1 atm pressure.
   (a) Is the pipe open or closed? **5 points**
   (b) What is the fundamental frequency? **5 points**
   (c) What is the length of the pipe? **5 points**
   (d) What is the expression $\xi(x,t)$ describing the fundamental? **5 points**
   (e) If the pipe were filled with monatomic helium, what would its fundamental frequency be? **5 points**
   (f) Compare and contrast the energy flow in standing waves and progressive waves and explain why they differ. **5 points**

a) "Open" pipes have harmonics that are integer multiples of the fundamental.
"Closed" pipes have harmonics that are odd integer multiples of the fundamental.
The harmonics of the organ pipe are 524 Hz apart, however, none of the harmonics are integer multiples of 524 Hz, so the harmonics must be spaces by $2\nu_0$, thus the pipe is closed and...

b) $\nu_0 = \frac{524\,Hz}{2} = 262\,Hz.$

c) for a closed pipe, the fundamental wavelength is $\lambda = 4L \Rightarrow L = \lambda/4$
Taking the speed of sound in air to be 343 m/s
$v = \lambda_0 \nu_0 \quad \lambda_0 = \frac{v}{\nu_0} = \frac{343\,m/s}{262\,s^{-1}} = 1.31\,m \qquad L = \frac{1.31\,m}{4} = 0.33\,m$

(parts a)-c) are from the April 4 WebAssign)

d) For a standing wave:

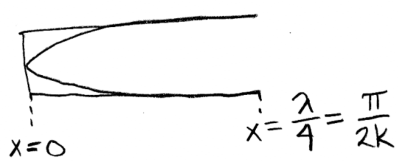
$\mathcal{E}(x,t) = 2\mathcal{E}_0 \sin kx \cos \omega t$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.31\,m} = 4.79\,m^{-1}$

$\omega = (2\pi)(262\,s^{-1}) = 1646\,s^{-1}$

$\mathcal{E}(x,t) = 2\mathcal{E}_0 \sin(4.79x)\cos(1646t)$

(assuming the closed end of the pipe corresponds to $x=0$)



function $\mathcal{E}$ must be 0 at $x=0$ and max at $x=\frac{\lambda}{4}$

e) In a gas: $C_w = \sqrt{\frac{\gamma P}{\rho_v}}$

$C_{He} = \sqrt{\frac{5}{3}\frac{(1\times10^5 N/m^2)}{(0.18\,kg/m^3)}} = 962\,m/s$

$\nu_0 = \frac{V}{\lambda_0} = \frac{962\,m/s}{1.31\,m} = 735\,Hz$

f) In progressive waves, energy flows (or is transported) along with the wave. There is no energy transport in standing waves since a standing wave is composed of two waves with equal energy traveling in opposite directions. The energy in a standing wave is therefore confined between the "nodes".

2. An ambulance speeds toward the facade of a large hospital at 20 m/s. It has a 50 W siren that emits sound isotropically with a frequency of 1000 Hz.

   (a) What is the intensity of the sound wave heard by the medical personnel waiting in front of the hospital when the ambulance is 200 m away? **5 points**

   (b) What is the displacement amplitude of the sound wave heard by the medical personnel at that instant? **5 points**

   (c) Write the general equation for the displacement wave as a function of the distance r from the siren. **5 points**

   (d) What frequency do the medical personnel standing in front of the hospital hear from the approaching siren? **5 points**

   (e) How many beats per second do the passengers in the ambulance hear due to the interference of the sound emitted from the siren and the sound reflected off the front of the hospital? **5 points**

a) $I = \frac{P}{A}$ (Eq. 7.3)

The spherical wavefronts cover an area of $4\pi r^2$.

$I = \frac{50W}{4\pi(200m)^2} = 1\times10^{-4} W/m^2$ or about 70 dB.

b) average intensity $= \frac{1}{2}\rho_r C_w \omega^2 \mathcal{E}_0^2$

$\mathcal{E}_0 = \frac{1}{\omega}\sqrt{\frac{2I}{\rho_r C_w}} = \frac{1}{2\pi(1000Hz)}\sqrt{\frac{2(1\times10^{-4} W/m^2)}{(1.29 kg/m^3)(343 m/s)}}$

$= 1.07\times10^{-7} m$

c) For spherical waves, the displacement wave is given by $\mathcal{E}(r,t) = \frac{A}{r}\sin(kr-\omega t)$ Eq. 7.9 ← where A is a constant

At 200m, $\mathcal{E}_0 = 1.07\times10^{-7}m$ $\quad \mathcal{E}_0 = \frac{A}{r}$

$\Rightarrow A = \mathcal{E}_0 r = (1.07\times10^{-7}m)(200m) = 2.14\times10^{-5}m^2$

The speed of sound in air is $343 m/s = \frac{\omega}{k}$

$\omega = 6280 s^{-1}$ $\quad k = \frac{6280 s^{-1}}{343 m/s} = 18.3 m^{-1}$

$\mathcal{E}(r,t) = \frac{5.7\times10^{-5}m^2}{r}\sin(18.3m^{-1}r - 6280s^{-1}t)$

d) In this case you have a moving source

Using Eq 8.7 $\nu' = \frac{C_s - U_o}{C_s - U_s} \nu_o$ U_o = speed of observer $= 0$

$U_s = 20 \, m/s$

U_s is positive since the siren is moving in the direction that the sound propogates from the siren to the observer.

$$\nu' = \frac{343 \, m/s}{343 \, m/s - 20 \, m/s} \nu_o = 1062 \, Hz$$

e) This is much like homework problem Chapter 8 #4. The hospital can be thought of as a stationary source emitting the Doppler shifted frequency, and the ambulance can be thought of as a moving observer.

ν_o becomes 1062 Hz $U_s = 0$ $U_o = -20 \, m/s$

why is U_o negative? Because the ambulance is moving toward the sound source $\xrightarrow[U_o]{}$ $\xleftarrow[C_s]{}$ so the sound and the observer are moving in opposite directions.

$$\nu' = \frac{C_s - U_o}{C_s - U_s} \nu_o = \frac{343 - (-20)}{343} 1062 \, Hz = 1124 \, Hz$$

so the beat frequency heard by the passengers in the ambulance $= (1124 - 1000) \, Hz = 124 \, Hz$

* Note! In fact, the intensity percieved by the people standing infront of the hospital is higher due to the Doppler shifted frequency. It makes a significant difference! But it's fine if you neglected this in parts a)-c).

3. An aluminum rod and a glass rod are joined at a smooth flat surface. A pulse in the glass rod is incident on the boundary.

   (a) What is the mechanical impedance of each rod? **5 points**
   (b) What is the speed of sound in each rod? **5 points**
   (c) What are the amplitudes of the transmitted and and reflected waves relative to the incident pulse? **5 points**
   (d) What fraction of the incident pulse energy is transmitted and what fraction is reflected? **5 points**

a) From Eq 6.28, $Z_m = \sqrt{\text{mass density} \times \text{Young's modulus}}$

$$Z_{glass} = \sqrt{(5.4\times10^{10}\,N/m^2)(2300\,kg/m^3)} = 1.11\times10^{7}\,kg/m^2\cdot s$$

$$Z_{aluminum} = \sqrt{(6.9\times10^{10}\,N/m^2)(2700\,kg/m^3)} = 1.36\times10^{7}\,kg/m^2\cdot s$$

b) From Eq. 5.20 $C_w = \sqrt{\frac{Y}{\rho_V}}$

$$C_{glass} = \sqrt{\frac{(5.4\times10^{10}\,N/m^2)}{(2300\,kg/m^3)}} = 4.8\times10^3\,m/s$$

$$C_{Al} = \sqrt{\frac{(6.9\times10^{10}\,N/m^2)}{(2700\,kg/m^3)}} = 5.1\times10^3\,m/s$$

c) $\mathcal{E}_r = \frac{Z_1 - Z_2}{Z_1 + Z_2}\mathcal{E}_1$ since the pulse starts in the glass, $Z_1 = Z_{glass}$

(Eq 6.29)

$$\mathcal{E}_r = \frac{1.11\times10^7\,kg/m^2\cdot s - 1.36\times10^7\,kg/m^2\cdot s}{1.11\times10^7\,kg/m^2\cdot s + 1.36\times10^7\,kg/m^2\cdot s}\,\mathcal{E}_1 = .1\,\mathcal{E}_1$$

Eq 6.27 $\mathcal{E}_2 = \frac{2Z_1}{Z_1+Z_2}\mathcal{E}_1 = \frac{2(1.11)}{1.11+1.36}\mathcal{E}_1 = 0.9\,\mathcal{E}_1$

Alternatively, you could just use eq. 6.25 $\mathcal{E}_2 = \mathcal{E}_1 + \mathcal{E}_r$ to get the transmitted wave amplitude.

d) From Eq 6.30

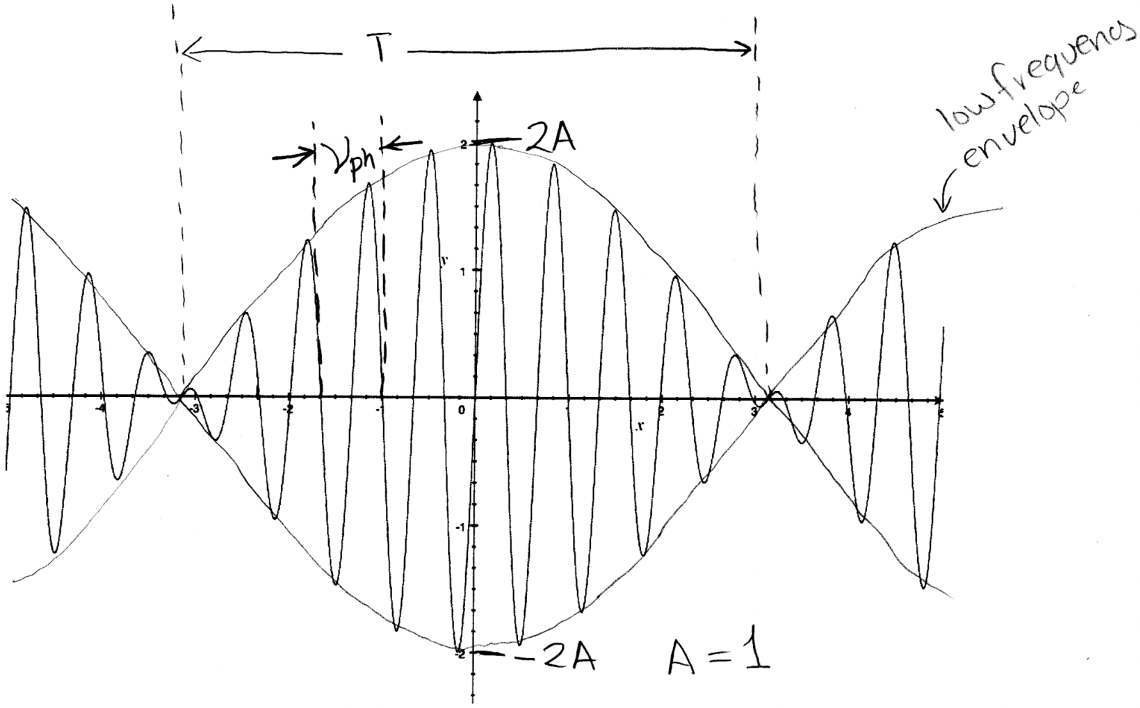
fraction of energy reflected $= \left(\frac{\mathcal{E}_r}{\mathcal{E}_i}\right)^2 = \frac{(0.1)^2 \cancel{\mathcal{E}_i^2}}{\cancel{\mathcal{E}_i^2}} = 0.01$

1% of energy is reflected

by conservation of energy, then 99% of energy is transmitted.

You might wonder why the amount of reflection is so small. It is because the impedances are very close to each other.

Parts a), c) and d) of this problem were chapter 6 #10 with different materials. You did that problem for homework.



4. The following refer to the above graph, which shows a waveform where the horizontal axis is in seconds.

   (a) Write Fourier's theorem in your own words. **5 points**

   (b) Identify the waveform above a write the constituent frequencies. **10 points**

   (c) Draw the fourier spectrum of the waveform. **5 points**

   (d) How long would you have to measure this waveform to capture all of the information (frequencies) contained within it? **5 points**

a) Any periodic function can be expressed as the sum of sinusoidal functions.

b) This is a beat pattern, which is formed when two waves with the same amplitude and slightly different frequencies are added together. (see Fig 2.18)

The beat period T is 6.28s. $\Delta\nu = \frac{1}{T} = \frac{1}{6.28s} = 0.16s^{-1}$

Eq 2.49 $|\nu_1 - \nu_2| = \Delta\nu = 0.16s$

The phase frequency is $T_{ph} = \frac{2}{3}s = \frac{\nu_1 + \nu_2}{2} \Rightarrow \nu_1 + \nu_2 = 3s^{-1}$

$\Rightarrow \nu_{ph} = \frac{3}{2}$

$$\nu_1 + \nu_2 = 3s^{-1}$$

$$\nu_1 - \nu_2 = 0.16s^{-1}$$

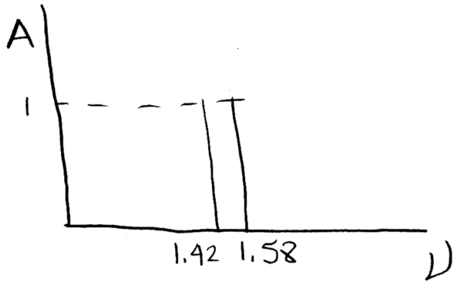
$$2\nu_1 = 3.16s^{-1}$$

$$\nu_1 = 1.58\,Hz$$

$$\nu_1 + \nu_2 = 3Hz \Rightarrow \nu_2 = 3Hz - \nu_1$$

$$\nu_2 = 3Hz - 1.58Hz = 1.42\,Hz$$

c)



d) You have to listen for 1 beat period, 6.28s. After that, the waveform just repeats itself.

Another way of looking at this is Eq. 13.10

pulse width x band with $\approx 2\pi$

$T \times \Delta\omega = 2\pi$ but $\Delta\omega = 2\pi\Delta f$

$T \times \Delta f = 1$

So you must listen $\frac{1}{\Delta f} = T$ to get the entire pulse.

Note: This problem was just like recitation 6 and one of the homework problems.

Possibly Useful Information

$\gamma = 7/5$ for diatomic gases (such as air)

$\gamma = 5/3$ for monatomic gases

for air: $\rho_v = 1.29$ kg/m^3

for Helium: $\rho_v = 0.18$ kg/m^3

1 atm $= 1 \times 10^5$ N/m^2

speed of sound in air at 20^o C and 1 atm =343 m/s

for aluminum: $Y = 6.9 \times 10^{10}$ N/m^2, $\rho_v = 2700$ kg/m^3

for glass: $Y = 5.4 \times 10^{10}$ N/m^2, $\rho_v = 2300$ kg/m^3