Name: ______________________________

I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

Signature: ______________________________

Make sure your exam has four pages . Continue on the back of the page if you run out of space. You may use one 4 × 6 recipe card with equations written on it. You may also use a calculator. Show all of your work.

PHYSICS 273 EXAM 1

1. A transverse sinusoidal wave on a taut wire has an amplitude of 2 mm and a wavelength of 1.22 m. The wave speed is 180 m/s. The wave is traveling in the positive x direction, and the displacement is in the y direction.

   (a) Write the function $y(x,t)$ describing the motion of the wave assuming that the displacement is at $x = 0$ and $t = 0$ is $y = 1$ mm and it is moving in the positive y direction. Give numerical values with units for all the parameters in $y(x,t)$. **10 points**

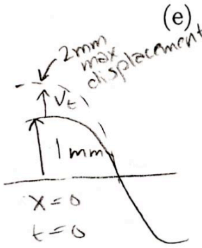
   (b) Find the value of the tension in the wire given a linear density of 2.45×10^{-2} kg/m. **5 points**

   (c) Find the maximum transverse speed of a point on the wire. **5 points**

   (d) Consider the position $x = 2.0$ m and find the first time after $t =$ 0at which the vertical speed of the wire is zero. **5 points**

   (e) Find the value of the *maximum* energy per unit length at any one position along the string. **5 points**

a)



$y = A\sin(kx - \omega t + \phi)$ $\quad k = \frac{2\pi}{\lambda} = 5.15\,m^{-1}$ $\quad v = \frac{\omega}{k}$

$\omega = v k = (5.15\,m^{-1})(180\,m/s) = 927\,Hz$

now we must use the initial conditions to find the phase shift ϕ

at $x=0$ $t=0$ $y=1mm$ and $dy/dt > 0$

$1mm = 2mm\sin\phi \Rightarrow \sin\phi = \frac{1}{2} \Rightarrow \phi = 30°$ or $150°$

because $dy/dt > 0$ $\quad \frac{dy}{dt} = -\omega A\cos(kx - \omega t + \phi)$

at $x=0$ $t=0$ $\frac{dy}{dt} = -\omega A\cos\phi \Rightarrow \cos\phi < 0$ so we choose $150°$ or $5\pi/6$

$y(x,t) = 0.002m\sin(5.15m^{-1}x - 927s^{-1}t + 5\pi/6)$

or equivalently

$y(x,t) = 0.002m\cos(5.15m^{-1}x - 927s^{-1}t + \pi/3)$

b) $v = \sqrt{\frac{T}{\rho_e}} \Rightarrow T = v^2\rho_e = (180\,m/s)^2(2.45\times10^{-2}\,kg/m) = 794N$

c) $v_{trans} = \frac{dy}{dt} = -\omega A\cos(kx - \omega t + \phi)$ so that $dy/dt|_{max} = -\omega A$

$= (927s^{-1})(0.002m) = 1.85\,m/s$

d) at $x = 2.0\text{m}$

$dy/dt = -\omega A \cos(k(2.0\text{m}) - \omega t + \phi) = 0$

$dy/dt = 0$ when $kx - \omega t + \phi$ is an odd multiple of $\pi/2$

$(5.15\text{m}^{-1}(2.0\text{m}) - 927\text{s}^{-1} t + \frac{5\pi}{6}) = \pm \frac{n\pi}{2}$: n odd

$12.92 - 927\text{s}^{-1} t = \pm n\pi/2$

$t = \frac{12.92 \mp 1.57n}{927\text{s}^{-1}} = \frac{12.92 - 1.57(7)}{927\text{s}^{-1}} = 2.08 \times 10^{-3}\text{s}$

$\frac{12.92}{1.57} = 8.2$ so for odd n t is minimized by choosing $n = 7$

(for $n = 1$, $t = 1.2 \times 10^{-2}\text{s}$)

e) $E/\Delta x = \rho_\ell v_{trans}^2 = (2.45 \times 10^{-2}\text{ kg/m})(1.85\text{m/s})^2 = 0.084\text{ J/m}$

$= \rho_\ell \omega^2 A^2$ see eq. 4.27

2. Suppose two waves traveling through a particular medium are superimposed:

$$y_1(x,t) = 0.02\sin(3x - 273t) \quad \text{and} \quad y_2(x,t) = 0.02\sin(4x - 364t)$$

where x and y are given in meters and t is in seconds.

(a) What is the beat frequency? **7 points**

(b) What is the group velocity? **6 points**

(c) Write the differential equation of motion of the superimposed wave. **7 points**

a) $\Delta f = \frac{\Delta\omega}{2\pi}$ Eq (2.49)

$\frac{364-273}{2\pi} = 14.48\,Hz$

b) the velocity of wave1 is $\frac{273}{3} = 91\,m/s$

velocity of wave2 is $\frac{364}{4} = 91\,m/s$

thus the group velocity of the superimposed wave is $91\,m/s$

c) Since y_1 and y_2 have the same velocity, $y_1 + y_2$ satisfies the wave equation with $c_w = 91\,m/s$

$$\boxed{\frac{d^2y}{dt^2} = (91\,m/s)^2 \frac{\partial^2 y}{\partial x^2}}$$

3. A coil with an inductance of 88mH and a 0.94 μF capacitor are connected in series with an alternating emf of frequency 930 Hz and an amplitude of 120V. The phase constant between the applied voltage and the current is found to be 75 degrees.

   (a) What is the resistance of the circuit? **7 points**

   (b) Draw a carefully labeled phasor diagram? **7 points**

   (c) If you could change the driving frequency, where would you tune it to make the circuit resonant? **6 points**

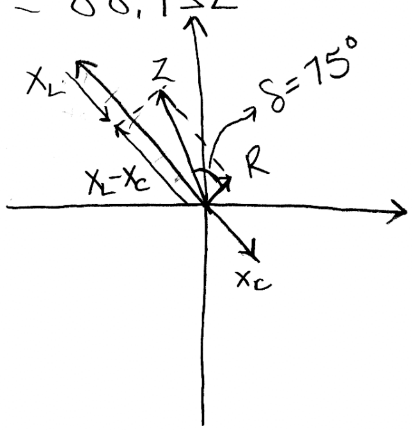
   (d) Draw as many parallels as you can between this system and a simple harmonic oscillator. Be as specific as possible. **10 points**

a) $\tan\delta = \frac{X_L - X_C}{R} \Rightarrow R = \frac{X_L - X_C}{\tan\delta}$ where $X_L = \omega L$, $X_C = 1/\omega C$, $\omega = 2\pi\nu$

$$R = \frac{\underbrace{(2\pi)(930s^{-1})(88\times10^{-3}H)}_{514\Omega} - \underbrace{1/(2\pi)(930s^{-1})(0.94\times10^{-6}F)}_{182\Omega}}{\tan 75^\circ}$$

$= 88.9\Omega$

b)



$X_L = 514\Omega$
$X_C = 182\Omega$
$R = 89\Omega$

c) Resonance at $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(88\times10^{-3}H)(0.94\times10^{-6}F)}}$

$\omega = 3.5\times10^{+3}$ rad/s

$\nu = \omega/2\pi = 553$ Hz

d) parent pushing a child on a swing
spring with a driving motor

from class notes

mechanical	electrical
displacement x	charge q
driving force F $W = F\,dx$	driving voltage V $W = V\,dq$
mass m (inertia resists acceleration)	inductance L (resists i when $di/dt \neq 0$)
viscous friction const. f (multiply by velocity to get the force)	resistance R (multiply by i to get the voltage drop)
spring constant k	reciprocal resistance $1/C$
(natural or resonant) frequency $\omega = \sqrt{\frac{k}{m}}$	(natural or resonant) frequency $1/\sqrt{LC}$
potential energy $\frac{1}{2}kx^2$	energy of static charge $\frac{1}{2}\frac{q^2}{C}$
kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$	electromagnetic energy of moving charge $\frac{1}{2}L\left(\frac{dq}{dt}\right)^2 = \frac{1}{2}Li^2$

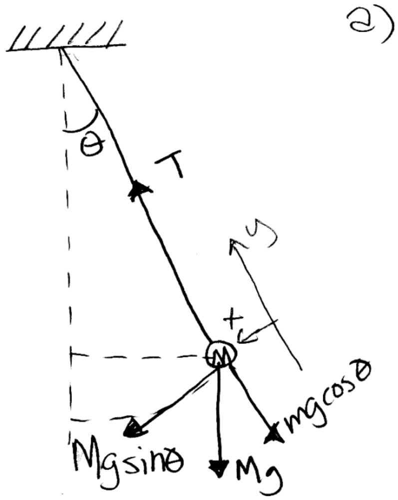
4. Consider an simple pendulum consisting of a mass M suspended from a massless string.

see pgs. 11–12 of H&L

(a) Draw a free body diagram of the system. What is the "restoring force"? **10 points**

(b) Write the equation of motion. Assume the oscillations are very small. **5 points**

(c) Solve the equation of motion using an exponential function. **5 points**



a) draw your axes so that one axis is parallel to the string and the other is along the direction of motion

$$\Sigma F_{axis1} = T - Mg\cos\theta = 0$$

$$\Sigma F_{axis2} = \underbrace{Mg\sin\theta}$$

this is along the direction of motion so it acts as the "restoring force"

b) eqn of motion $F = ma$

$$\cancel{M}\frac{d^2x}{dt^2} = -\cancel{M}g\sin\theta$$

for circular motion distance traveled $= \ell\theta$

$$\ell\frac{d^2\theta}{dt^2} = -g\sin\theta$$

for small θ $\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\sin\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\theta = 0$$

c) Try a solution of the form $\theta = \theta_0 e^{i\omega t}$ $\quad \frac{d^2\theta}{dt^2} = -\omega^2\theta_0 e^{i\omega t}$

$$-\omega^2\cancel{\theta_0}\cancel{e^{i\omega t}} + \frac{g}{\ell}\cancel{\theta_0}\cancel{e^{i\omega t}} = 0 \qquad \omega^2 = \frac{g}{\ell} \qquad \omega = \sqrt{\frac{g}{\ell}}$$

so the solution is $\theta = \theta_0 e^{i\sqrt{\frac{g}{\ell}}t}$

only the real part describes the motion so:

$$\theta = \theta_0 \operatorname{Re}\left(e^{i\sqrt{g/\ell}\,t}\right) = \theta_0\cos\sqrt{\tfrac{g}{\ell}}\,t$$