Phys 273 – Formula Sheet #2

$$\ddot{y}_p + 2\omega_0^2 y_p - \omega_0^2(y_{p+1} + y_{p-1}) = 0, \ \ \omega_0^2 = T/ml, \rightarrow \vec{x}(t) = \sum_{n=1}^{N} a_n \vec{q}_n e^{i\omega_n t}$$

$$\vec{q}_n = \left(\sin\left(\frac{n\pi}{N+1}\right), \sin\left(\frac{2n\pi}{N+1}\right), \ldots, \sin\left(\frac{Nn\pi}{N+1}\right) \right), \ A_{pn} = \sin\left(\frac{pn\pi}{N+1}\right), \ a_i = \frac{\vec{y}_0 \cdot \vec{q}_i}{|\vec{q}_i|^2}$$

$$\omega_n = 2\omega_0 \sin\left(\frac{n\pi}{2(N+1)}\right)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T}\frac{\partial^2 y}{\partial t^2}, \ v = \sqrt{T/\rho}$$

$$y(x,t) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{L}\right) e^{i\omega_n t}, \ \omega_n = \sqrt{\frac{T}{\rho}}\frac{n\pi}{L}, \ c_n = \frac{2}{L}\int_0^L y(x,t=0)\sin\left(\frac{n\pi x}{L}\right) dx$$

$$y(x,t) = A\sin(kx - \omega t), \ k = \frac{2\pi}{\lambda}, \ \omega = 2\pi f = \frac{2\pi}{T}, \ v = \lambda f = \frac{\omega}{k}$$

$$y(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk A(k) e^{i(kx+\omega t)}, \ A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx f(x) e^{-ikx}$$

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right) dx = \frac{L}{2}\delta_{nm}, \ \sum_{j=1}^{N} \sin\left(\frac{jn\pi}{N+1}\right)\sin\left(\frac{jm\pi}{N+1}\right) = \left(\frac{N+1}{2}\right)\delta_{nm}$$

$$Z = \frac{F}{\dot{y}} = \frac{T}{v_p} = \rho v_p$$

$$\frac{A_2}{A_1} = \frac{2Z_1}{Z_1 + Z_2}, \ \frac{B_1}{A_1} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$P = \frac{1}{2}Z\omega^2 A^2$$

$$v = \frac{1}{\sqrt{L_0 C_0}}, \ Z_0 = \sqrt{\frac{L_0}{C_0}}$$

$$\frac{V_-}{V_+} = \frac{Z_L - Z_0}{Z_L + Z_0}, \ \frac{V_L}{V_+} = \frac{2Z_L}{Z_L + Z_0}, \ \frac{I_-}{I_+} = \frac{Z_0 - Z_L}{Z_0 + Z_L}, \ \frac{I_L}{I_+} = \frac{2Z_0}{Z_L + Z_0}$$

$$y(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk A(k) e^{i(kx - \omega(k)t)}$$

$$v_p \equiv \frac{\omega(k)}{k}, \ v_g = \frac{d\omega}{dk}$$

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$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right],$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \; b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/L}, \; c_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{-in\pi x/L} dx,$$

$$a_n = c_n + c_{-n}, \; b_n = i(c_n - c_{-n}), \; a_0 = 2c_0$$

$$c_n = \begin{cases} \frac{1}{2}(a_{-n} + ib_{-n}), & n < 0 \\ \frac{1}{2}a_0, & n = 0 \\ \frac{1}{2}(a_n - ib_n), & n > 0 \end{cases}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk\, A(k) e^{ikx}, \; A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, f(x) e^{-ikx}$$