Phys 273 - Homework #8

1) Consider the function

$$y(x) = \begin{cases} 0 & -L < x < -1/2 \\ d & -1/2 < x < 1/2 \\ 0 & 1/2 < x < L \end{cases}$$, and repeating with period 2L.

In this expression, (d) is a constant, and L > 1.

a) Sketch this function for the case L = 1, assuming d = 1.

b) Sketch this function for the case L =4, assuming d = 1.

c) Consider L = 1 and d = 1 again. Calculate the expansion coefficients for the complex Fourier Series for this function.

d) Let q = nπ. Re-write the expansion coefficients from part (c) as a function of (q), and make a plot of the expansion coefficients as a function of (q).

e) Now we let L go to infinity, so that the function is no longer periodic. Sketch the function again and calculate its Fourier Transform, A(k).

f) Plot the Fourier Transform A(k) as a function of k.

g) Compare the result from part (d) to the result from part (f). What are the similarities, and what are the differences?

2) A transverse wave on an infinitely long string is described by

$$y(x,t) = 0.5\sin\left(\frac{\pi x}{2} - 50\pi t\right)$$

a) What are the amplitude, wavelength, and wave number of the wave?

b) What are the frequency (f), period, and velocity of the wave?

c) What is the maximum transverse speed of any particle in the string.

3) A wave on a string with a frequency of 20 Hz travels with a velocity of 80 m/s.

a) If the mass density of the string is 0.1 kg/m, what is the tension of the string?

b) What is the distance between two points on the wave which have a phase difference of 30 degrees?

4) Recall problem #2 of homework 7, where we calculated the Fourier Series for this square wave:

$$f(x) = \begin{cases} -1, -L < x < 0 \\ 1, 0 < x < L \end{cases}, \text{ periodic with period 2L.}$$

The result was

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin\left(\frac{n\pi x}{L}\right), \text{ odd (n) only}$$

Let's re-calculate this Fourier Series again, but this time let's use the complex form of the series:

$$f(x) = \sum_{n=-\infty}^{n=\infty} c_n e^{in\pi x/L}$$

Fourier's trick tells us that the coefficients $\{c_n\}$ can be calculated according to:

$$c_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{-in\pi x/L} dx$$

a) Calculate (c_0), for this square.
b) Calculate the rest of the Fourier coefficients $\{c_n\}$ for this square wave.
c) Since this f(x) is purely real, the coefficients $\{c_n\}$ should have the property that

$$c_n = c_{-n}^*$$

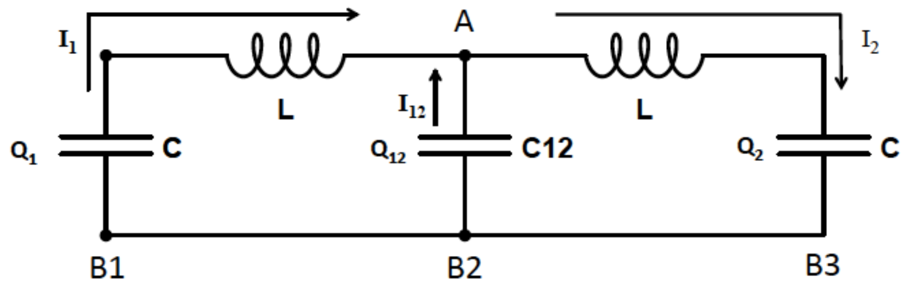
Check to see if this is true using your result from part (b).

d) The real coefficients $\{a_n\}$ and $\{b_n\}$ that you calculated on homework #7 should be related to the complex coefficients from parts (a) and (b) according to:

$$a_n = c_n + c_{(-n)}$$
$$b_n = i(c_n - c_{(-n)})$$
$$a_0 = 2c_0$$

Check to see if this is true.

5) Consider two coupled LC oscillators:



This circuit is the electrical analog of the coupled mechanical oscillators that we studied earlier in the semester (see Homework #5).

a) To analyze this circuit, start by rewriting the known equations of motion for the coupled mechanical oscillators in the following form:

$$\ddot{x}_1 + \omega_0^2(1+\alpha)x_1 - \omega_0^2\alpha x_2 = 0$$
$$\ddot{x}_2 + \omega_0^2(1+\alpha)x_2 - \omega_0^2\alpha x_1 = 0$$

Identify the constants (ω_0) and (α) in terms of (k), (k_{12}), and (m), and write down the two normal mode frequencies for the mechanical oscillator in terms of (ω_0) and (α). (You don't need to solve the equation of motion to do this; just write the known expressions for (ω_1) and (ω_2) in terms of (ω_0) and (α)).

b) Consider the three currents (I_1, I_2, and I_{12}) which meet at junction A in the above circuit diagram. Use Kirchoff's current sum rule to write down the relationship between these currents. Note the sign convention indicated in the diagram: the direction of each arrowhead gives the direction of positive current.

c) Use the relationship between the currents and the charges in the circuit to convert the current sum rule from part (b) into a sum rule for the charges (Q_1, Q_2, and Q_{12}).

d) Consider the voltage drop from junction A to junctions B_1, B_2, and B_{12}. Since these three voltage drops must be the same, we have $V(AB_1) = V(AB_2)$ and $V(AB_3) = V(AB_2)$ Using the voltage drop rules for capacitors and inductors, write these two equations in terms of the currents, charges, L, C, and C_{12}.

e) Use the charge sum rule from part (c) to eliminate (Q_{12}) from the two voltage drop equations.

f) Take the time derivative of both equations to get the coupled equations of motion for (I_1) and (I_2). Show that these equations of motion can be written in the same form as the mechanical oscillator equations from part (a), and identify the constants (ω_0) and (α) for the coupled LC oscillators.