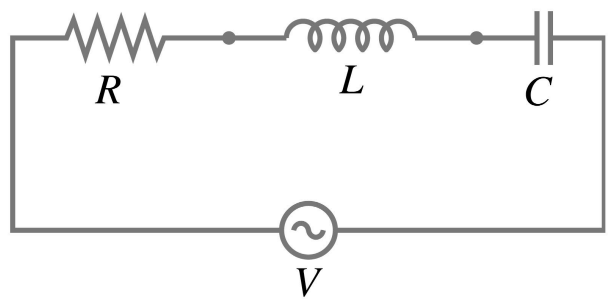
Physics 273 - Homework #5

1) **Series RLC circuit.**

Consider a series RLC circuit driven by a voltage source:



By considering the phasor diagram for the voltages in this circuit, we found the following expression for the circuit impedance:

$$\left|Z_{series}\right| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\text{ , } \omega = \text{driving frequency.}$$

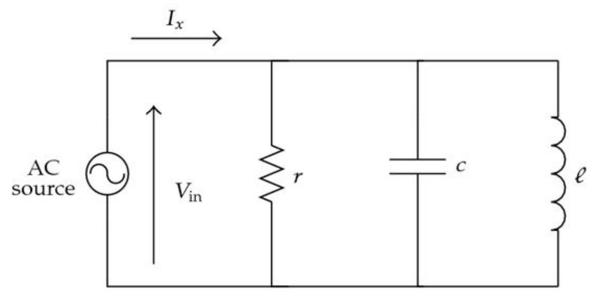
a) Show that you can deduce this expression for the impedance <u>without a phasor diagram</u> simply by using the rules for combining impedances in series.

b) Suppose that the magnitude of the voltage source is one volt, the inductance is 1 milli-Henry, and the capacitance is one milli-Farad. Use a plotting program to make a three plots of the **peak current as a function of driving frequency** for three resistance values: R = 0.025 Ohms, R = 0.1 Ohms, and R = 0.4 Ohms. <u>Please plot the peak current value for driving frequencies (ω) ranging from 2 Hz to 2,000 Hz, and please put the three curves on the same plot, so we can compare your curves by eye.</u>

c) Please label your three curves according to the resistance value of each. Qualitatively speaking, what is the difference between the three curves?

d) Does this current in this circuit have a resonance behavior? If so, what resistance value gives the highest Q factor, and which resistance value gives the smallest Q factor?

2) **Parallel RLC circuit.** Here is a driven RLC circuit where the circuit elements are connected in parallel:



a) How many independent voltages are there in this circuit?

b) Draw a phasor diagram for this circuit. Start by drawing the voltage phasor for the voltage source, and then add three current phasors, one each for the resistor, capacitor, and inductor.

c) There is a fourth current phasor in the circuit, the current delivered by the voltage source. We can determine its relationship to the other three current phasors using Kirchoff's current sum rule. Write down the current sum rule for this circuit as an equation, and draw it as a phasor diagram.

d) Using the rules for combining impedances in parallel, show that the impedance for this circuit can be written as

$$Z_{para} = \frac{R\omega L}{\omega L + i\left(\omega^2 RLC - R\right)},$$ where ω is the driving frequency.

e) Prove the following identity for complex numbers: If $z = \dfrac{a}{b + ic}$, then $|z| = \dfrac{a}{\sqrt{b^2 + c^2}}$.

f) Use the identity from part (e) to find an expression for the magnitude of Z_{para} as a function of the driving frequency, R, L, and C.

g) Suppose that the magnitude of the voltage source is one volt, the inductance is 1 milli-Henry, and the capacitance is one milli-Farad. Use a plotting program to make a three plots of the **peak current as a function of driving frequency** for three resistance values: R = 0.1 Ohms, R = 1.0 Ohms, and R = 10.0 Ohms. Please plot the peak current value for driving frequencies (ω) ranging from 2 Hz to 20,000 Hz, and please put the three curves on the same plot, so we can compare your curves by eye.

Note: In question (1b), I asked you to plot up to ω = 2,000 Hz, whereas in this question I have asked you to plot up to 20,000 Hz.

h) Does this current in this circuit have a resonance behavior? If so, what resistance value gives the highest Q factor, and which resistance value gives the smallest Q factor?

3) **Ringing of a mechanical oscillator and critical damping (numerical).** Make a copy of your numerical solution to Homework #4 problem #2 (forced oscillator with damping). In this problem we will change the forcing function to the following step function (or square function):

$$F(t) = \begin{cases} 1.0 \text{ Newtons}, & 0 < t < 10 \text{ s} \\ 0.0 \text{ Newtons}, & 10 \text{ s} < t < 20 \text{ s} \end{cases}$$

The forcing function repeats itself thereafter with a period of 20 seconds. Hint: If you are using excel, one way to implement this forcing function is to create a new column which contains the value of the forcing function at each moment in time. You can then reference this column when calculating the acceleration.

a) Let x0 = 0.0 m, v0 = 0.0 m/s, m = 1 kg, k = 30 N/m, and b = 2 N/(m/s). Use a time step of 0.01 seconds, and calculate for 4000 steps (a total of 40 seconds), and plot the position of the oscillator as a function of time. In your solution you should see a phenomena called "ringing". Measure the frequency of the ringing, and compare it to the oscillation frequency that you would expect for this oscillator.

b) Print out a plot of your solution from part (a).

c) The frequency of a damped oscillator is given by

$$\omega_d = \sqrt{\omega_0^2 - \gamma^2/4}$$

Suppose we increase the drag coefficient (b or γ) until $\omega_d = 0.0$. This condition is called "critical damping", and it is the condition for all oscillations to be eliminated. Calculate the value of the drag coefficient (b) which achieves critical damping for the oscillator parameters described in part (a).

d) Starting with a drag coefficient of b = 2 N/(m/s), increase (b) one unit at a time up to 20 N/(m/s), and observe the effect on the oscillator's position as a function of time. Now set (b) equal to the critical value that you calculated in part (c), and print out a plot of the oscillator's position for this situation.

Comment: In real mechanical and electrical systems where oscillations are undesirable, the components will often be chosen so that the system is critically damped. For example, a building or a bridge might be designed to be critically damped to prevent large oscillations in the event of an earthquake.

e) When the drag coefficient is increased beyond the critical value, we say that the system is "over-damped". Just like critically damped systems, over-damped systems also do not oscillate, however, they take longer to return to equilibrium after a shock. To see the response of an over-damped system, set b = 50 N/(m/s), and print out a plot of the oscillator's position as a function of time.