UNIVERSITY OF MARYLAND, PHYSICS DEPARTMENT

PHYS 272, HW 1 Solutions

Jonathan Echevers

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1 PROBLEM A

1.1 PART I

The poker player start with some unknown amount of money, lets call it x. Then he makes \$100 on the first game, \$200 on the second and looses \$50 on the last. Therefore, the total amount of cash available to the player at the end of her playing will be $x+100+200-50 = x-250$, as such, the difference in her cash position will be \$250

Suppose $c(t) = 3e^{-2t^2}$
So:

$$\int_{t_a}^{t_b} \frac{d}{dt}c(t)dt = \int_{t_a}^{t_b} \frac{d}{dt}3e^{-2t^2})dt$$
$$= 3\int_{t_a}^{t_b} e^{-2t^2}(-4t)dt = 3e^{-2t^2}|_{t_a}^{t_b}$$
$$= 3(e^{-2t_b^2} - e^{-2t_a^2})$$
$$= c(t_b) - c(t_a)$$

1.2 PART II

See Fig. 1.1 in last page.

To find the largest value of x for which the difference between $sinx$ and $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ be less than 1% set:

$$sinx - x + \frac{x^3}{3!} - \frac{x^5}{5!} > \frac{1}{100}$$

Which yields $-1.76 < x < 1.76$. You can "solve" the previous equation by entering it in your favorite numerical solver (e.g. Matlab, Matematica, etc.) For Matlab, the following code suffices: syms x; solve(sin(x)-x+x^3/factorial(3)-x^5/factorial(5)-1/100)

2 Problem B

2.1 Part I

$$\mathbf{A} - 3\mathbf{B}$$
$$= (-3-6)\hat{x} + 2 + 3\hat{y} + (1-12)\hat{z}$$
$$= -9\hat{x} + 5\hat{y} - 11\hat{z}$$

Here, $\hat{x}, \hat{y}, \hat{z}$ denote the unit vectors in the coordinate system x, y, z. Clearly, the substraction produces a new vector with components in the x, y and z axes.

2.2 Part II

$$\mathbf{A} \cdot \mathbf{B}$$
$$= -6 - 2 + 4 = -4$$

2.3 Part III

See Fig 2.1 for geometrical interpretation. Denote the projection of $\mathbf{B}$ onto $\mathbf{A}$ as the vector $\mathbf{B_1}$, then:

$$|\mathbf{B_1}| = |\mathbf{B}| cos\theta$$

Where $|\mathbf{A}|$ denotes the magnitude of the vector and θ denotes the angle between $|\mathbf{A}|$ and $|\mathbf{B}|$. Then:

$$\mathbf{B_1} = \alpha \mathbf{A}$$

Where α is a scalar constant that, when multiplied by the vector $\mathbf{A}$ produces the vector $\mathbf{B_1}$.

$$|\mathbf{B_1}| = |\mathbf{A}|\alpha$$
$$\alpha = \frac{|\mathbf{B_1}|}{|\mathbf{A}|} = \frac{\mathbf{B}}{\mathbf{A}} cos\theta$$
$$\text{So: } \mathbf{B_1} = \frac{\mathbf{B} \cdot \mathbf{A}}{\mathbf{A} \cdot \mathbf{A}} \mathbf{A}$$

2.4 Part IV

$$|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| sin\theta$$

Where θ denotes the angle between $\mathbf{A}$ and $\mathbf{B}$.

$$\text{So: } \theta = sin^{-1} \frac{|\mathbf{A} \times \mathbf{B}|}{|\mathbf{A}||\mathbf{B}|}$$

2.5 Part v

$$\mathbf{A}\times\mathbf{B}$$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -3 & 2 & 1 \\ 2 & -1 & 4 \end{vmatrix}.$$

$$= (8+1)\hat{x} - (-12-2)\hat{y} + (3-4)\hat{z}$$
$$= 9\hat{x} + 14\hat{y} - \hat{z}$$

The new vector produced by the cross product will be orthogonal (perpendicular) to both vectors **A** and **B**.

2.6 Part vi

The solution is most easy to see if you think of the vectors **A** and **B** as a triangle (call it triangle A), when connected by a third vector that goes to and/or from each extrema. Then think of this triangle as composed of two smaller **right** triangles, such that if you add up their areas, you get the total area of the triangle A (recall that the area of a right triangle is just one half times its base times height.) Now, if you think of the area of the parallelogram produced by the vectors **A** and **B**, it should be clear that its just twice the area of triangle A.
So:

$$Area = |\mathbf{A}||\mathbf{B}|sin\theta = |\mathbf{A}\times\mathbf{B}|$$

It should now be very easy to see how to prove Part IV (just write out the cross product explicitly!)

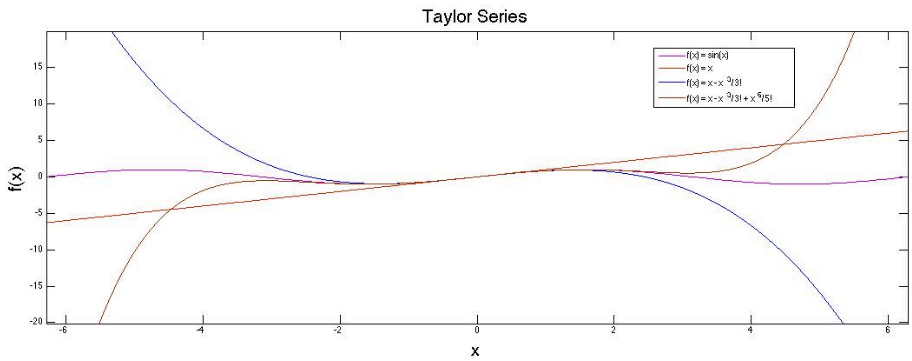

Figure 1.1: Taylor Series. Notice: as we add more terms to the series, the approximation becomes a better fit for the sin(x) function.

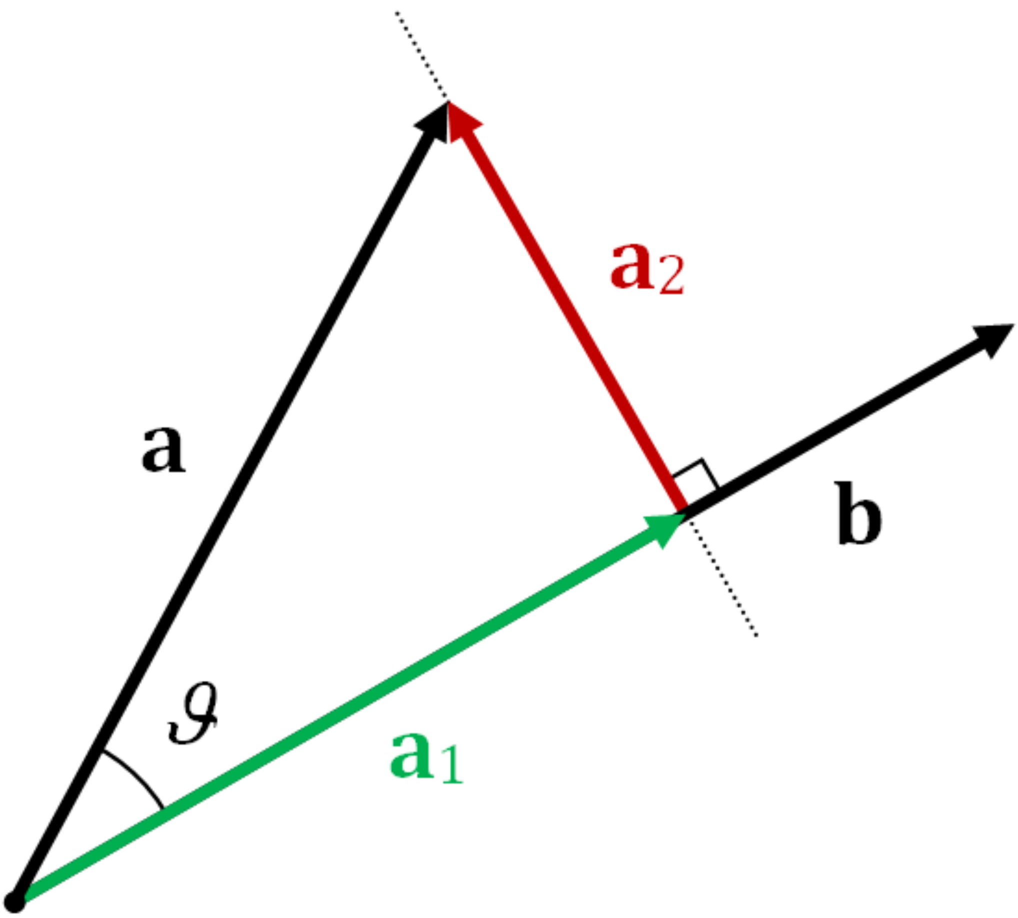

Figure 2.1: Geometrical interpretation of vector projection: here, the projection of an arbitrary vector **a** onto another vector (call it **b**) is denoted by the new vector $\mathbf{a_1}$