PHY 272: FIELDS

PROBLEM SET 1

due February 4, **before** *class*

A. Idiotic calculus refresher

i) A poker player makes \$100 on the first game, \$200 on the next and looses \$50 on the one after that. What is the difference between her cash position between the beginning and the end of her playing?

Now, let us make a "continuous time" model of the situation. Call $c(t)$ the amount of money the player is holding at time t. She wins (or looses) money at a *rate* given by $dc(t)/dt$ so, over an infinitesimal time period dt, her cash position changes by $dc(t)/dtdt$. Between instants t_a and t_b the change in her cash position will be the sum over these infinitesimal changes

$$\int_{t_a}^{t_b} \frac{dc(t)}{dt} dt = c(t_b) - c(t_a). \tag{1}$$

This is the "Fundamental Theorem of Calculus". Verify it explicitly, by computing both sides of the equation above, in the case $c(t) = 3e^{-2t^2}$.

ii) We will frequently approximate expressions when a certain parameter is "small" using its Taylor series. You probably remember the general formula

$$f(x) \approx f(x_0) + (x - x_0)\frac{df}{dx}|_{x=x_0} + \frac{(x-x_0)^2}{2!}\frac{d^2 f}{dx^2}|_{x=x_0} + \cdots . \tag{2}$$

Just as useful is to remember a few special cases by heart. They are:

$$\frac{1}{1-x} \approx 1 + x + x^2 + \cdots , \tag{3}$$

$$\log(1-x) \approx -x - \frac{x^2}{2} + \cdots , \tag{4}$$

$$e^x \approx 1 + x + \frac{x^2}{2!} + \cdots , \tag{5}$$

$$\sin x \approx x - \frac{x^3}{3!} + \cdots , \tag{6}$$

$$\cos x \approx 1 - \frac{x^2}{2!} + \cdots , \tag{7}$$

$$\tag{8}$$

Notice that the second formula can be obtained by integrating both sides of the first one and the fourth and fifth are the odd and even parts of the third. Also notice that these formuale are useful when x is small because in this case we can truncate them at the expense of a small error. Write these five formulae in a piece of paper and tape it to the mirror in your bathroom. Also, plot on the same graph $\sin x$, x, $x - x^3/3!$ and $x - x^3/3! + x^5/r!$ and notice how these expressions approximate $\sin x$ better and better. How large can x be so the difference between $\sin x$ and $x - x^3/3! + x^5/r!$ is larger than 1%?

B. A few things about vectors

Let $\mathbf{A} = -3\hat{\mathbf{x}} + 2\hat{\mathbf{y}} + \hat{\mathbf{z}}$ and $\mathbf{B} = 2\hat{\mathbf{x}} - \hat{\mathbf{y}} + 4\hat{\mathbf{z}}$ ($\hat{\mathbf{x}}$, ... are the unit vectors on the directions x, ...). Compute

i) $\mathbf{A} - 3\,\mathbf{B}$

ii) $\mathbf{A}.\mathbf{B}$

iii) The projection of $\mathbf{B}$ on the $\mathbf{A}$ direction

iv) The angle between $\mathbf{A}$ and $\mathbf{B}$

v) $\mathbf{A} \times \mathbf{B}$

vi) the area of the parallelogram with sides given by $\mathbf{A}$ and $\mathbf{B}$