UNIVERSITY OF
MARYLAND

Solution : #3 corrected

Department of Physics
Physics 270 Fall 2008

Dr. Wallace, MWF 12:00, Room 1410 Physics

Sections: 0301 (W 1:00); 0302 (W 2:00); 0303 (Th 10:00)

Midterm Exam. I Monday October 13, 2008

Name (please print):

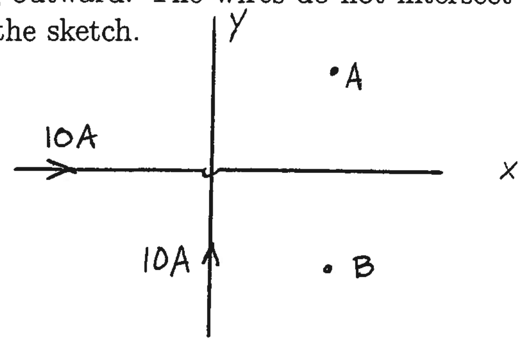
Circle section number above. Exams will be returned in sections.

Pledge: I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

Signature:

Useful constants: $\mu_0 = 4\pi \times 10^{-7}$ Tm/A, $\epsilon_0 = 8.85 \times 10^{-12}$ $C^2/N\ m^2$

1.) Two infinite wires are in the plane of the page, one running along the x-axis and the other along the y-axis. The z-axis is perpendicular to the page, with the unit vector $\hat{z}$ pointing outward. The wires do not intersect and each wire carries a current of 10 A as shown in the sketch.



a.) (10 pts.) Calculate the magnetic field **vector** at point A, which is a distance 5 cm from each wire.

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\left[\hat{z} - \hat{z}\right] = 0$$

b.) (10) pts.) Calculate the magnetic field **vector** at point B, which is a distance 5 cm from each wire.

$$\vec{B} = \frac{4\pi \times 10^{-7}\,\frac{Tm}{A} \times 10A}{2\pi(.05\,m)}\left(-\hat{z} - \hat{z}\right) = -800 \times 10^{-7}\,T\,\hat{z}$$

2.) An electromagnetic wave has its electric field in the y-direction. It is described by

$$E_y(x,t) = 20 sin(10^7 x - 3 \times 10^{15} t) \quad V/m \tag{1}$$

a.) (5 points) What is the wavelength of the light?

$$k = 10^7\,m^{-1} = \frac{2\pi}{\lambda}, \quad \lambda = 2\pi \times 10^{-7}\,m = 628\,nm.$$

b.) (5 points) What is the frequency of the light?

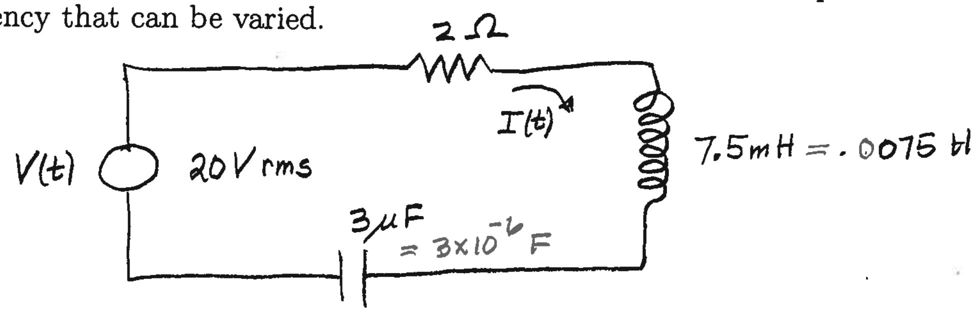
$$f = \frac{\omega}{2\pi} = \frac{3 \times 10^{15}\,1/sec}{2\pi} = .477 \times 10^{15}\,Hz$$

c.) (10 points) What is the intensity of the light in W/m^2?

$$S_{ave} = \left(\frac{1}{2}\epsilon_0 E_0^2\right)c = \frac{1}{2}\,8.85 \times 10^{-12}\,\frac{C^2}{Nm}\left(20\frac{V}{m}\right)^2\left(3 \times 10^8\,m/s\right)$$

$$= 0.534\,\frac{W}{m^2}$$

3 ~~4~~.) A circuit as shown below is driven by an oscillator that produces 20 V rms at a frequency that can be varied.



a.) (10 points) Apply Kirchoff's rule to this circuit to obtain an equation that relates the current I(t) to the driving voltage V(t).

$$V(t) - 2\,I(t) - .0075\,\frac{dI(t)}{dt} - \frac{Q(t)}{3\times10^{-6}} = 0$$

where $$\frac{dQ(t)}{dt} = I(t)$$

b.) (5 points) What is the resonance frequency of the circuit in Hz?

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{.0225\times10^{-6}}} = 6667\ \frac{\text{rad}}{\text{sec}}, \qquad f = \frac{\omega}{2\pi} = 1061\ \text{Hz}$$

c.) (5 points) What is the rms current in the circuit when the oscillator drives it at the resonance frequency?

At resonance, $Z = R = 2\,\Omega$, $\quad I_{rms} = \frac{V_{rms}}{Z} = \frac{20\,V}{2\,\Omega} = 10\,A$

d.) (5 points) What is the impedance of the circuit when the driving frequency is 1000 Hz? $\omega = 2\pi f = 6283$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{4 + \left(.0075\times6283 - \frac{1}{3\times10^{-6}\times6283}\right)^2}$$

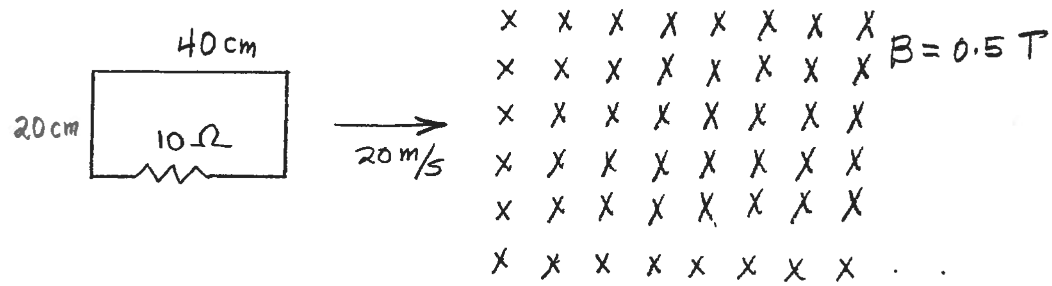
$$= \sqrt{4 + (47.12 - 53.0)^2}$$

$$= 6.26\ \Omega$$

5.) (5 points) Circle the name of the person who showed that light is an electromagnetic wave.

Ampere Faraday Gauss Maxwell Weber Henry Watt Tesla

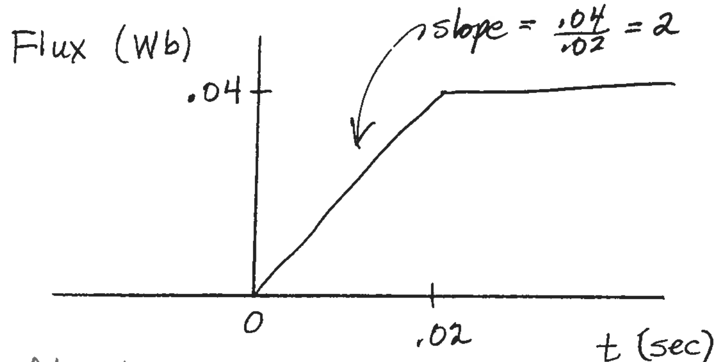
4.) A 20 cm by 40 cm rectangular loop is made of conducting wire and a 10 Ω resistor. This loop is moving at constant speed 20 m/s toward the right and it enters a region with a uniform magnetic field of 0.5 Tesla pointing into the page (shown by the x's). Assume a sharp boundary between the region to the left where there is no field and the region that has the 0.5 Tesla field.



a.) (10 pts.) Calculate the time it takes the loop to enter the field region completely. Then plot the flux that passes through the square loop versus time, t, with t=0 being the time the loop enters the magnetic field. Show on your plot the maximum value of the flux and the time when the loop is inside.

$$t = \frac{.40\,m \text{ (width)}}{20\,m/s \text{ (velocity)}} = .02\,sec$$

$$\Phi_{max} = B \cdot A = (.2m)(.4m)(.5T) = .04\,Tm^2 = .04\,Wb$$



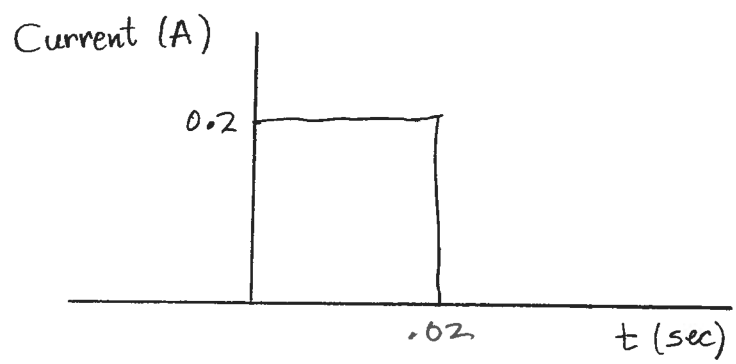
$\Phi = 0$ before $t=0$; Φ = constant after $t = .02\,sec$

b.) (10 pts.) Plot the current through the resistor as a function of time, identifying the numerical value of the maximum current on your plot.

$$\mathcal{E} = -\frac{d\Phi}{dt} = 0 \quad \text{unless } 0 < t < .02\,sec.$$

$$\mathcal{E} = -\frac{d}{dt}\left(\frac{.04\,Wb}{.02\,sec} \cdot t\right) = -2V$$

$$I = 2V/10\,\Omega = 0.2A$$



c.) (5 pts.) What is the direction of the current in the loop

Circle one: clockwise counterclockwise

d.) (5 points) Is there a force on the loop at any time? If so what is the direction of the force?

Yes – it is to the left – against the motion of the loop when $0 < t < 0.02$ sec. [Lenz's Law]