HW 7 Solutions

35.2 $\Delta x = ct$; $c = \frac{\Delta x}{t} = \frac{2(1.50 \times 10^8 \text{ km})(1000 \text{ m/km})}{(22.0 \text{ min})(60.0 \text{ s/min})} = 2.27 \times 10^8 \text{ m/s} = \boxed{227 \text{ Mm/s}}$

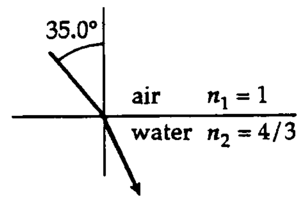
35.3 The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next: $t = 2\ell/c$

$$\theta = \omega t = \omega\left(\frac{2\ell}{c}\right) \qquad \text{so} \qquad \omega = \frac{c\theta}{2\ell} = \frac{(2.998 \times 10^8)[2\pi/(720)]}{2(11.45 \times 10^3)} = \boxed{114 \text{ rad/s}}$$

The returning light would be blocked by a tooth at one-half the angular speed, giving another data point.

35.5 Using Snell's law, $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$

$$\theta_2 = \boxed{25.5°} \qquad \lambda_2 = \frac{\lambda_1}{n_1} = \boxed{442 \text{ nm}}$$

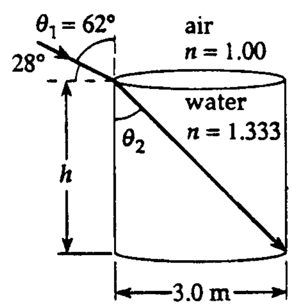


***35.13** $\sin\theta_1 = n_w \sin\theta_2$

$$\sin\theta_2 = \frac{1}{1.333}\sin\theta_1 = \frac{1}{1.333}\sin(90.0° - 28.0°) = 0.662$$

$$\theta_2 = \sin^{-1} 0.662 = 41.5°$$

$$h = \frac{d}{\tan\theta_2} = \frac{3.00 \text{ m}}{\tan 41.5°} = \boxed{3.39 \text{ m}}$$



***35.16** At entry, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ or $1.00 \sin 30.0° = 1.50 \sin\theta_2$

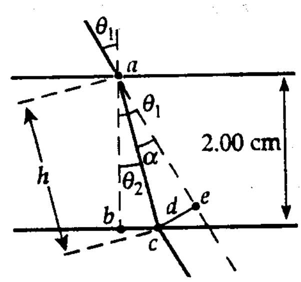
$$\theta_2 = 19.5°$$

The distance h the light travels in the medium is given by

$$\cos\theta_2 = \frac{(2.00 \text{ cm})}{h} \qquad \text{or} \qquad h = \frac{(2.00 \text{ cm})}{\cos 19.5°} = 2.12 \text{ cm}$$

The angle of deviation upon entry is $\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$

The offset distance comes from $\sin\alpha = \frac{d}{h}$: $d = (2.21 \text{ cm})\sin 10.5° = \boxed{0.388 \text{ cm}}$

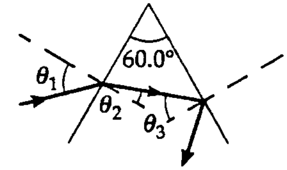


35.27 Taking Φ to be the apex angle and δ_{min} to be the angle of minimum deviation, from Equation 35.9, the index of refraction of the prism material is

$$n = \frac{\sin\left(\frac{\Phi + \delta_{min}}{2}\right)}{\sin(\Phi/2)}$$

Solving for δ_{min}, $\delta_{min} = 2\sin^{-1}\left(n \sin\frac{\Phi}{2}\right) - \Phi = 2\sin^{-1}[(2.20)\sin(25.0°)] - 50.0° = \boxed{86.8°}$

35.31 At the first refraction, $(1.00)\sin\theta_1 = n\sin\theta_2$



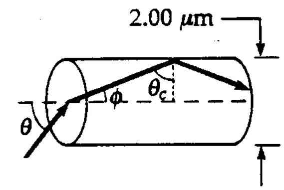
The critical angle at the second surface is given by

$$n\sin\theta_3 = 1.00, \quad \text{or} \quad \theta_3 = \sin^{-1}\left(\frac{1.00}{1.50}\right) = 41.8°.$$

But, $\theta_2 = 60.0° - \theta_3$. Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3 < 41.8°$), it is necessary that $\theta_2 > 18.2°$. Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes

$$\sin\theta_1 > (1.50)\sin(18.2°) = 0.468, \quad \text{or} \quad \theta_1 > \boxed{27.9°}$$

***35.38** $\sin\theta_c = \dfrac{n_{\text{air}}}{n_{\text{pipe}}} = \dfrac{1.00}{1.36} = 0.735 \qquad \theta_c = 47.3°$



Geometry shows that the angle of refraction at the end is

$$\theta_r = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$$

Then, Snell's law at the end, $\quad 1.00\sin\theta = 1.36\sin 42.7°$

gives $\quad \boxed{\theta = 67.2°}$

35.39 For total internal reflection, $\quad n_1\sin\theta_1 = n_2\sin 90.0°$

$(1.50)\sin\theta_1 = (1.33)(1.00) \qquad$ or $\qquad \theta_1 = \boxed{62.4°}$

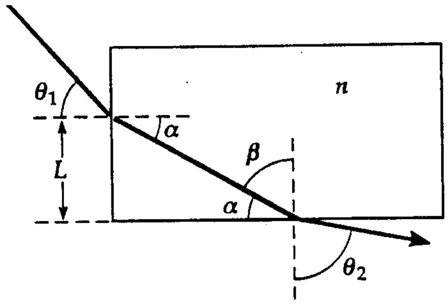
***35.43** For plastic with index of refraction $\boxed{n \geq 1.42}$ surrounded by air, the critical angle for total internal reflection is given by

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) \leq \sin^{-1}\left(\frac{1}{1.42}\right) = 44.8°$$

In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from both the sides of the slab and from facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark. To frustrate total internal reflection in the gasoline, the index of refraction of the plastic should be $\boxed{n < 2.12}$, since

$$\theta_c = \sin^{-1}\left(\frac{1.50}{2.12}\right) = 45.0°.$$

35.69 (a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$, Snell's law at the first surface gives



$$n \sin\alpha = (1.00)\sin 45.0° \qquad (1)$$

Observe that the angle of incidence at the second surface is $\beta = 90.0° - \alpha$. Thus, Snell's law at the second surface yields

$$n \sin\beta = n \sin(90.0° - \alpha) = (1.00)\sin 76.0°, \text{ or}$$

$$n \cos\alpha = \sin 76.0° \qquad (2)$$

Dividing Equation (1) by Equation (2), $\tan\alpha = \dfrac{\sin 45.0°}{\sin 76.0°} = 0.729$ or $\alpha = 36.1°$

Then, from Equation (1), $n = \dfrac{\sin 45.0°}{\sin\alpha} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$

(b) From the sketch, observe that the distance the light travels in the plastic is $d = L/\sin\alpha$. Also, the speed of light in the plastic is $v = c/n$, so the time required to travel through the plastic is

$$t = \frac{d}{v} = \frac{nL}{c \sin\alpha} = \frac{(1.20)(0.500 \text{ m})}{(3.00\times10^{8} \text{ m/s})\sin 36.1°} = 3.40\times10^{-9} \text{ s} = \boxed{3.40 \text{ ns}}$$