Homework 12 Solutions

40.7 (a) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(620\times10^{12}\ \text{s}^{-1})\left(\dfrac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$

(b) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.10\times10^{9}\ \text{s}^{-1})\left(\dfrac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.28\times10^{-5}\ \text{eV}}$

(c) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(46.0\times10^{6}\ \text{s}^{-1})\left(\dfrac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.91\times10^{-7}\ \text{eV}}$

(d) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{620\times10^{12}\ \text{Hz}} = 4.84\times10^{-7}\ \text{m} = \boxed{484\ \text{nm, visible light (blue)}}$

$\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{3.10\times10^{9}\ \text{Hz}} = 9.68\times10^{-2}\ \text{m} = \boxed{9.68\ \text{cm, radio wave}}$

$\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{46.0\times10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m, radio wave}}$

40.9 Each photon has an energy $E = hf = (6.626\times10^{-34})(99.7\times10^{6}) = 6.61\times10^{-26}\ \text{J}$

This implies that there are $\dfrac{150\times10^{3}\ \text{J/s}}{6.61\times10^{-26}\ \text{J/photons}} = \boxed{2.27\times10^{30}\ \text{photons/s}}$

40.15 (a) $\lambda_c = \dfrac{hc}{\phi} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(4.20\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = \boxed{296\ \text{nm}}$

$f_c = \dfrac{c}{\lambda_c} = \dfrac{3.00\times10^{8}\ \text{m/s}}{296\times10^{-9}\ \text{m}} = \boxed{1.01\times10^{15}\ \text{Hz}}$

(b) $\dfrac{hc}{\lambda} = \phi + e(\Delta V_S)$: $\dfrac{(6.626\times10^{-34})(3.00\times10^{8})}{180\times10^{-9}} = (4.20\ \text{eV})(1.60\times10^{-19}\ \text{J/eV}) + (1.60\times10^{-19})(\Delta V_S)$

Therefore, $\boxed{\Delta V_S = 2.71\ \text{V}}$

40.18 From condition (i), $hf = e(\Delta V_{S1}) + \phi_1$ and $hf = e(\Delta V_{S2}) + \phi_2$

$(\Delta V_{S1}) = (\Delta V_{S2}) + 1.48\ \text{V}$

Then $\phi_2 - \phi_1 = 1.48\ \text{eV}$

From condition (ii), $hf_{c1} = \phi_1 = 0.600hf_{c2} = 0.600\phi_2$

$\phi_2 - 0.600\phi_2 = 1.48\ \text{eV}$

$\boxed{\phi_2 = 3.70\ \text{eV}}$ $\boxed{\phi_1 = 2.22\ \text{eV}}$

40.20 From the photoelectric equation, we have: $e(\Delta V_{S1}) = E_{\gamma 1} - \phi$ and $e(\Delta V_{S2}) = E_{\gamma 2} - \phi$

Since $\Delta V_{S2} = 0.700(\Delta V_{S1})$, then $e(\Delta V_{S2}) = 0.700(E_{\gamma 1} - \phi) = E_{\gamma 2} - \phi$

or $(1 - 0.700)\phi = E_{\gamma 2} - 0.700\,E_{\gamma 1}$

and the work function is: $\phi = \dfrac{E_{\gamma 2} - 0.700\,E_{\gamma 1}}{0.300}$

The photon energies are: $E_{\gamma 1} = \dfrac{hc}{\lambda_1} = \dfrac{1240\text{ nm}\cdot\text{eV}}{410\text{ eV}} = 3.03\text{ eV}$

and $E_{\gamma 2} = \dfrac{hc}{\lambda_2} = \dfrac{1240\text{ nm}\cdot\text{eV}}{445\text{ eV}} = 2.79\text{ eV}$

Thus, the work function is $\phi = \dfrac{2.79\text{ eV} - 0.700(3.03\text{ eV})}{0.300} = 2.23\text{ eV}$

and we recognize this as characteristic of $\boxed{\text{potassium}}$.

40.38 (a) $\lambda_{min} = \dfrac{hc}{E_{max}}$

Lyman ($n_f = 1$): $\lambda_{min} = \dfrac{hc}{|E_1|} = \dfrac{1240\text{ eV}\cdot\text{nm}}{13.6\text{ eV}} = \boxed{91.2\text{ nm}}$ (Ultraviolet)

Balmer ($n_f = 2$): $\lambda_{min} = \dfrac{hc}{|E_2|} = \dfrac{1240\text{ eV}\cdot\text{nm}}{\left(\frac{1}{4}\right)13.6\text{ eV}} = \boxed{365\text{ nm}}$ (UV)

Paschen ($n_f = 3$): $\lambda_{min} = \ldots = 3^2(91.2\text{ nm}) = \boxed{821\text{ nm}}$ (Infrared)

Bracket ($n_f = 4$): $\lambda_{min} = \ldots = 4^2(91.2\text{ nm}) = \boxed{1460\text{ nm}}$ (IR)

(b) $E_{max} = \dfrac{hc}{\lambda_{min}}$

Lyman: $E_{max} = \boxed{13.6\text{ eV}}$ $(= |E_1|)$

Balmer: $E_{max} = \boxed{3.40\text{ eV}}$ $(= |E_2|)$

Paschen: $E_{max} = \boxed{1.51\text{ eV}}$ $(= |E_3|)$

Brackett: $E_{max} = \boxed{0.850\text{ eV}}$ $(= |E_4|)$

40.66 $\Delta\lambda = \dfrac{h}{m_p c}(1 - \cos\theta) = \dfrac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})}{(1.67\times 10^{-27}\text{ kg})(3.00\times 10^{8}\text{ m/s})}(0.234) = 3.09\times 10^{-16}\text{ m}$

$\lambda_0 = \dfrac{hc}{E_0} = \dfrac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(3.00\times 10^{8}\text{ m/s})}{(200\text{ MeV})(1.60\times 10^{-13}\text{ J/MeV})} = 6.20\times 10^{-15}\text{ m}$

$\lambda' = \lambda_0 + \Delta\lambda = 6.51\times 10^{-15}\text{ m}$

(a) $E_\gamma = \dfrac{hc}{\lambda'} = \boxed{191\text{ MeV}}$

(b) $K_p = \boxed{9.20\text{ MeV}}$

40.31 (a) Thanks to Compton we have four equations in the unknowns ϕ, v, and λ':

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + \gamma m_e c^2 - m_e c^2 \qquad \text{(energy conservation)} \qquad [1]$$

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'} \cos 2\phi + \gamma m_e v \cos \phi \qquad \text{(momentum in } x \text{ direction)} \qquad [2]$$

$$0 = \frac{h}{\lambda'} \sin 2\phi - \gamma m_e v \sin \phi \qquad \text{(momentum in } y \text{ direction)} \qquad [3]$$

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos 2\phi) \qquad \text{(Compton equation)} \qquad [4]$$

Using $\sin 2\phi = 2 \sin \phi \cos \phi$ in Equation [3] gives $\gamma m_e v = \frac{2h}{\lambda'} \cos \phi$.

Substituting this into Equation [2] and using $\cos 2\phi = 2\cos^2\phi - 1$ yields

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'}(2\cos^2\phi - 1) + \frac{2h}{\lambda'}\cos^2\phi = \frac{h}{\lambda'}(4\cos^2\phi - 1),$$

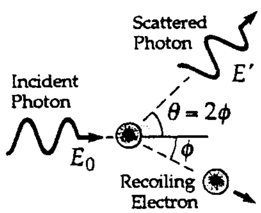
or $\lambda' = 4\lambda_0 \cos^2\phi - \lambda_0$ [5]

Substituting the last result into the Compton equation gives

$$4\lambda_0\cos^2\phi - 2\lambda_0 = \frac{h}{m_e c}\left[1 - \left(2\cos^2\phi - 1\right)\right] = 2\frac{hc}{m_e c^2}\left(1 - \cos^2\phi\right).$$

With the substitution $\lambda_0 = hc/E_0$, this reduces to

$$\cos^2\phi = \frac{m_e c^2 + E_0}{2m_e c^2 + E_0} = \frac{1+x}{2+x} \quad \text{where} \quad x \equiv \frac{E_0}{m_e c^2}.$$



For $x = \frac{0.700 \text{ MeV}}{0.511 \text{ MeV}} = 1.37$, this gives $\phi = \cos^{-1}\sqrt{\frac{1+x}{2+x}} = \boxed{33.0°}$

(b) From Equation [5], $\lambda' = \lambda_0\left(4\cos^2\phi - 1\right) = \lambda_0\left[4\left(\frac{1+x}{2+x}\right) - 1\right] = \lambda_0\left(\frac{2+3x}{2+x}\right)$.

Then, Equation [1] becomes

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda_0}\left(\frac{2+x}{2+3x}\right) + \gamma m_e c^2 - m_e c^2 \quad \text{or} \quad \frac{E_0}{m_e c^2} - \frac{E_0}{m_e c^2}\left(\frac{2+x}{2+3x}\right) + 1 = \gamma.$$

Thus, $\gamma = 1 + x - x\left(\frac{2+x}{2+3x}\right)$, and with $x = 1.37$ we get $\gamma = 1.614$.

Therefore, $\frac{v}{c} = \sqrt{1 - \gamma^{-2}} = \sqrt{1 - 0.384} = 0.785$ or $v = \boxed{0.785c}$.

40.42 $\Delta E = (13.6\text{ eV})\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$

Where for $\Delta E > 0$ we have absorption and for $\Delta E < 0$ we have emission.

(A) for $n_i = 2$ and $n_f = 5$ $\Delta E = 2.86$ eV (absorption)

(B) for $n_i = 5$ and $n_f = 3$ $\Delta E = -0.967$ eV (emission)

(C) for $n_i = 7$ and $n_f = 4$ $\Delta E = -0.572$ eV (emission)

(D) for $n_i = 4$ and $n_f = 7$ $\Delta E = 0.572$ eV (absorption)

(a) $E = \frac{hc}{\lambda}$ so the shortest wavelength is emitted in transition $\boxed{B}$.

(b) The atom gains most energy in transition $\boxed{A}$.

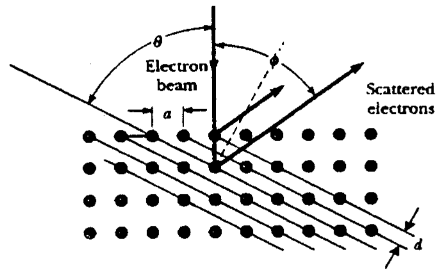
(c) The atom loses energy in transitions $\boxed{\text{B and C}}$.

40.56 From the Bragg condition (Eq. 38.13),

$$m\lambda = 2d\sin\theta = 2d\cos(\phi/2)$$

But, $d = a\sin(\phi/2)$ where a is the lattice spacing. Thus, with $m = 1$,

$$\lambda = 2a\sin(\phi/2)\cos(\phi/2) = a\sin\phi$$



$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\text{ kg})(54.0\times1.60\times10^{-19}\text{ J})}} = 1.67\times10^{-10}\text{ m}$$

Therefore, the lattice spacing is

$$a = \frac{\lambda}{\sin\phi} = \frac{1.67\times10^{-10}\text{ m}}{\sin 50.0°} = 2.18\times10^{-10}\text{ m} = \boxed{0.218\text{ nm}}$$

40.63 $$K_{\max} = \frac{q^2B^2R^2}{2m_e} = \frac{(1.60\times10^{-19}\text{ C})^2(2.00\times10^{-5}\text{ T})^2(0.200\text{ m})^2}{2(9.11\times10^{-31}\text{ kg})} = 2.25\times10^{-19}\text{ J} = 1.40\text{ eV} = hf - \phi$$

$$\phi = hf - K_{\max} = \frac{hc}{\lambda} - K_{\max} = \frac{(4.14\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{450\times10^{-9}\text{ m}} - 1.40\text{ eV} = \boxed{1.36\text{ eV}}$$