Homework 4:

Remember: In addition to this problem, you also have a "Mastering Physics" assignment Due February 27. This assignment is due at 10:00 A.M Friday, February 27 otside my office. Write up of the solution to this problem in a coherent fashion.

In class we showed that the circumstances for which sinusoidal wave solved all four of Maxwell equations. There are of course also solutions which do not correspond to pure sine waves lasting for ever---after all there can be pulses of light. Tt can be shown that following form also solves Maxwell's equations:

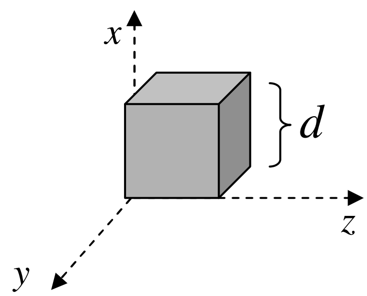
$$\vec{E} = \hat{x}\frac{A}{\sqrt{L^2 + (z-ct)^2}} \qquad \vec{B} = \hat{y}\frac{A}{c\sqrt{L^2 + (z-ct)^2}}$$

where A is a constant (with dimensions of voltage) and L is a constant (with dimensions of length) . These fields describe a pulse of radiation propagating in the z direction. This problem concerns the energy and energy flow of this electromagnetic wave pulse.

a. Show that the electromagnetic energy density, $u_{E-M} = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$, is given by $u_{E-M} = \frac{\varepsilon_0 A^2}{L^2 + (z-ct)^2}$.

b. Show that the Poynting vector (which describes the energy flow per unit time per unit area in the direction of propagation) defined by $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$ is given by $\vec{S} = \hat{z}\frac{\varepsilon_0 c A^2}{L^2 + (z-ct)^2}$

This problem considers the electromagnetic energy inside a cube and the rate at which energy enters (or leaves) it. The cube has sides is aligned with the x, y and z axis; is corner is at the origin as in the figure below:



c. Show that total electromagnetic energy inside the cube at any given time is given by $U_{EM} = d^2\int_0^d u_{EM}(z,t)\,dz = \frac{d^2 A^2 \varepsilon_0}{L}\left(\arctan\left((d-ct)/L\right) + \arctan\left(ct/L\right)\right)$. A helpful mathematical fact: $\int_0^d \frac{1}{L^2+(z-b)^{\wedge}2}dz = \left(\arctan((d-b)/L) + \arctan(b/L)\right)/L$

d. Use $\vec{S}$ to show that the power carried by the electromagnetic wave entering the cube through the face a *z=0* (the one hidden in the figure) is given by $P_{entering} = \frac{\varepsilon_0 c A^2 d^2}{L^2 + (ct)^2}$ and the power leaving the cube through the face a *z=d* (the dark one in the figure) is given by $P_{leaving} = \frac{\varepsilon_0 c A^2 d^2}{L^2 + (d-ct)^2}$.

e. Use parts c. and d. to show that $\frac{dU_{EM}}{dt} = P_{entering} - P_{leaving}$. Explain why this makes sense on physical grounds.