Physics 270 Final Exam

You may use a one-page (both sides) formula sheet for this exam. You may use a calculator---but it should not be necessary. No other written or electronic aids are allowed. The exam ends promptly at 3:30.

In problems calling for expressions as answers, give your expressions in terms of the parameters of the problem.

Show your work. To get full credit you must indicate how you obtained your answers from the physical principles studied in the course. Moreover, to receive partial credit it is essential that you make your reasoning clear. For some questions I will provide the answer (which may be useful in later sections) and ask you to explain how the answer is obtained. To get any credit for these you MUST show your reasoning---after all, you already have the correct answer.

Work the problems on the exam sheet. If you need extra space, please use the pages at the back of the exam (front and back) and label the problem number. If you do use a page in the back please indicate that you have done so.

You will be provided with a sheet of scratch paper; you can use this to check calculations before writing them down.

Name:

Section:

Honor Pledge:
"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination."

Signature:

1. A new planet named Zircon has been discovered. It is located 16 light-years from earth around a nearby star. There is a plan to accelerate a spacecraft (named the Zirconian Falcon) to a very high speed---4/5 *c*---in a very short time (which for the purpose of this problem you can take to be zero) and to have it coast to Zircon. In the earth's reference frame it will take 20 years to reach Zircon. The Zirconian Falcon is 50 m long in its rest frame.

a. From the perspective of an astronaut on the Zirconian Falcon, how much time will have elapsed from launch till landing on Zircon?

b. From the earth's frame of reference, how long (in meters) is the Zirconian Falcon when it is cruising?

Consider two events: event 1 is when the Zirconian Falcon leaves earth and event 2 is when it gets to Zircon. From the earth's perspective these events are given by the following space time points: event 1 is at $(t_1 = 0, x_1 = 0, y_1 = 0, z_1 = 0)$ and event 2 is at $(t_2 = 20 \text{ years}, x_2 = 16 \text{ light- years}, y_2 = 0, z_2 = 0)$, where the line between earth and Zircon is taken to be in the x direction. The space-time coordinates in the frame moving with the Zirconian Falcon are denoted with primes; the position of the Zirconian Falcon is $x'=0$. In this frame event 1 is at $(t_1' = 0, x_1' = 0, y_1' = 0, z_1' = 0)$ and event 2 $(t_2' = \tau, x_2' = 0, y_2' = 0, z_2' = 0)$, where τ is the answer to part a. From general relativistic considerations, the space-time interval between the two events (defined by $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - c^2(t_1 - t_2)^2$) as measured in the two frames are related.

c. What is this relationship?

d. Verify that this relation in c is satisfied given the numbers in this problem.

2. Consider a beam of laser light. Over a large region of space, it is described by a running electromagnetic wave moving in the positive x direction. The electric field is polarized in the z direction; it is given by $\vec{E} = \hat{k}\, E_0 \sin\big(k(x - c\,t)\big)$. *(Note, this is not exactly like the case in the book where the E field is polarized in the y direction.)*

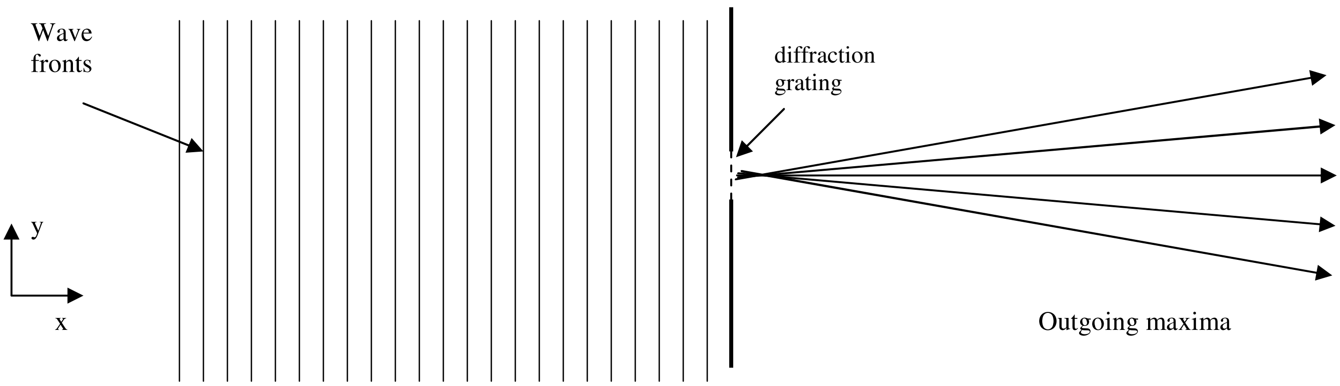
a. Show that the frequency of this wave is given by $f = \dfrac{ck}{2\pi}$.

b. Write an expression for the magnetic field. (Hint: What is its magnitude? In what direction is it pointing? In figuring out the direction, it may help to recall that the Poynting vector $\vec{S}$ points in the direction of propagation.)

c. Find an expression for the time-averaged energy density for this wave in terms of the parameters of the problem.

d. Find the (time-averaged) density of photons in this wave (that is the number of photons per unit volume)

e. The wave is directed towards a diffraction grating as in the following figure below. The grating has slits separated by a distance *d*. Find the angles of the maxima emerging from this grating in terms of the parameters of the problem.



3. An electron starting at rest is accelerated in an electric field to a speed of $\frac{12}{13}c$. The mass of the electron is denoted m_e and its charge is $-e$.

a. Find the magnitude of the difference of the electrostatic potential (that is the voltage) across this electric field. Express your answer in terms of m_e, c, and *e*.

b. Find an expression for the momentum of the electron after it has been accelerated.

c. What is the de Broglie wavelength of the electron after it has been accelerated? (Give your answer in terms of *h, c,* m_e and e).

4. This problem concerns the photon and focuses on the fact that it must have a mass of zero. Consider a photon associated with radiation with frequency *f*.

a. Determine the energy and momentum of the photon using the standard quantum relations. Give your answers in terms of *f* and universal constants such as *h* and *c*.

b. Show that these values of *E* and *p,* together with relativity, requires that the mass be zero.

5. Electromagnetic radiation in the form of monochromatic (single frequency) ultra-violet light is shined on a hydrogen atom which is in its ground state. When the frequency of this radiation is above some minimum the radiation can ionize the hydrogen: that is it can knock the electron completely out of the atom leaving behind a bare proton and a free electron moving away at some velocity. It is helpful to think about this process in a photon picture in which the electron is knocked out by a single photon and to consider the implications of conservation of energy. This is very much like the photo-electric effect in a metal except that instead of the emitted electron coming out with a range of energies (the maximum of which is given by the Einstein formula) the electron is emitted with a single energy which is determined by the frequency of the light and properties of hydrogen.

In doing this problem the following fact may be helpful. The energy of the hydrogen atom in its n^{th} level is accurately given by the Bohr formula $E_n = -\frac{B}{n^2}$ where $n = 1, 2, 3, 4, \ldots$, and *B*, the binding energy of the ground state is given by $B = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{m_e}{2\hbar^2} \approx 13.6\,eV$; m_e is the mass of the electron and -*e* is its charge.

a. Show that when an electron is knocked out of the ground-state (n=1) hydrogen atom, the kinetic energy of the outgoing electron is given by $KE = -B + hf$.

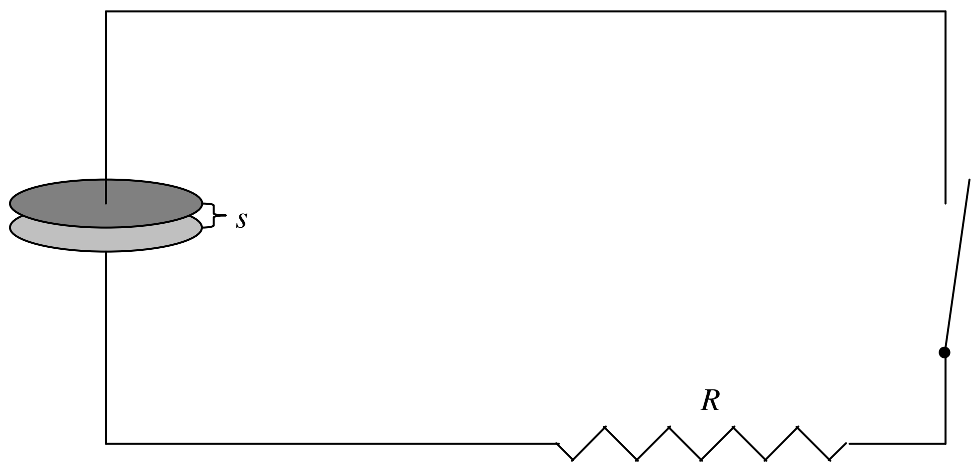
b. What is the minimum frequency needed to knock the electron out of the ground-state hydrogen? Explain *very briefly* why this makes sense.

c. Find the de Broglie wavelength of the electron after it has been ionized. Express your answer in terms of the parameters in this problem. (In doing this problem you may assume that the electron is moving slowly enough so that the KE is given by its nonrelativistic formula of $KE = \frac{1}{2} m_e v^2$).

d. Suppose instead of the ultraviolet radiation shining on a hydrogen atom in its ground state, it is shined on a hydrogen atom in its n=3 level and knocks the electron out of it (ionizes it). Find a formula for the energy of the knocked out electron for this case (*i.e.* the analog of the formula in part *a.* appropriate for this case.)

6. A parallel plate capacitor is made from two coaxial disks each with area A which are separated by a distance *s*. You may assume that *s* is small enough so that edge effects are negligible. At time *t=0* there is a charge of Q_0 on the top plate and -Q_0 on the bottom plate. The plates are connected to a switch and a resistor of resistance *R* as in the figure below. At time *t=0* the switch is thrown and the capacitor discharges through the resistor.

A useful fact: the capacitance of a parallel plate capacitor in the regime where edge effects are negligible is given by $C = \frac{\varepsilon_0 A}{s}$ where *A* is the area of the plates and *s* is the separation.



If you cannot get a section of this problem relax and go on to the next section. The problem is written in such a way if you cannot get one section you still do the subsequent sections.

a. Use Kirchkoff's loop law to show that the charge on the upper plate satisfies the following differential equation $\frac{sQ}{\varepsilon_0 A} + R\frac{dQ}{dt} = 0$.

b. From the result of a. show that

i. The voltage across the plates for times $t>0$ is given by

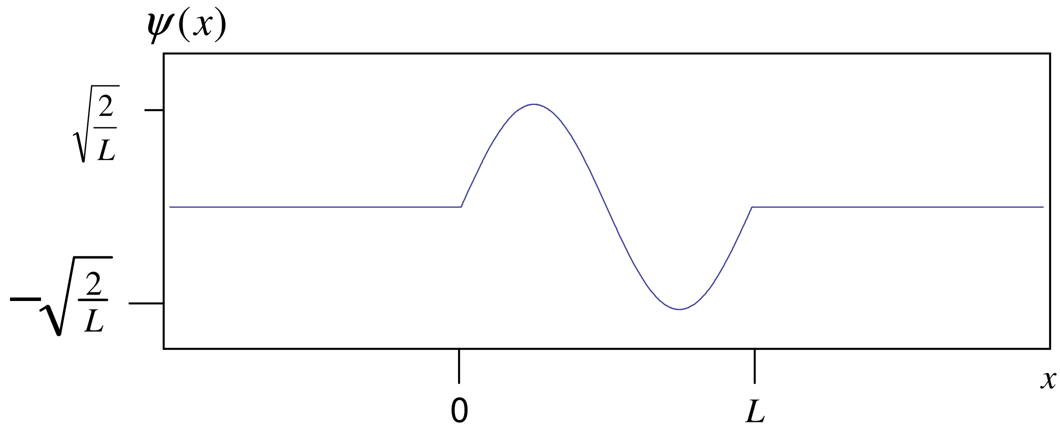
$$V = \frac{s\,Q_0}{\varepsilon_0 A}\exp\left(-\frac{s}{\varepsilon_0 AR}t\right).$$

ii. The electric field between the plates is directed downward and has a magnitude of $E = \frac{Q_0}{\varepsilon_0 A}\exp\left(-\frac{s}{\varepsilon_0 AR}t\right)$.

c. Show that between the capacitor plates the Maxwell displacement current density is directed upward and has an amplitude of $J_d = \frac{Q_0 s}{\varepsilon_0 A^2 R}\exp\left(-\frac{s}{\varepsilon_0 AR}t\right)$

d. This displacement current causes a magnetic field. If the wires are long enough so that the magnetic fields from them can be neglected, symmetry implies that the magnetic field between the plates will be of the form $\vec{B}(\vec{r},t) = \hat{\theta}\,B(r,t)$ where $\hat{\theta}$ is a unit vector directed around the axis and $B(r,t)$ only depends on *r,* the distance from the axis, and time. Find $B(r,t)$ in terms of Q_0,s, *A, R,* and fundamental constants such as ε_0.

7. The second allowed energy state for a one-dimension box (located from 0 to L) has a normalized wave function given by $\psi(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right) & \text{for} \quad 0 \le x \le L \\ 0 & \text{for} \quad x > L \end{cases}$. This is shown below.



Some of the following integrals might be helpful in doing this problem:

$$\int_a^b dx \sin^2\left(\tfrac{2\pi x}{L}\right) = \tfrac{1}{2}(b-a) + \tfrac{L}{8\pi}\left(\sin\left(\tfrac{4\pi a}{L}\right) - \sin\left(\tfrac{4\pi b}{L}\right)\right) \qquad \int_0^L dx \sin^2\left(\tfrac{2\pi x}{L}\right) x = \frac{L^2}{4}$$

$$\int_0^L dx \sin\left(\tfrac{2\pi x}{L}\right)\cos\left(\tfrac{2\pi x}{L}\right) = 0 \qquad \int_0^L dx \sin^2\left(\tfrac{2\pi x}{L}\right) x^2 = L^3\left(\frac{1}{6} - \frac{1}{16\pi^2}\right) \qquad \int_0^L dx \sin\left(\tfrac{2\pi x}{L}\right)\cos\left(\tfrac{2\pi x}{L}\right) x = -\frac{L^2}{8\pi}$$

a. What is the probability that the particle is located in the region $\frac{L}{4} \le x \le \frac{3L}{4}$?

b. For this wave function show that the expectation value (average values) x and x^2 are given by:

$$\langle x \rangle = \frac{L}{2}$$

$$\langle x^2 \rangle = L^2\left(\frac{1}{3} - \frac{1}{8\pi^2}\right)$$

c. Calculate Δx , the uncertainty in x.

d. In this problem you have not computed the uncertainty in the momentum for this wave function. However given what you already have computed and general principles, it is possible to deduce that the uncertainty in p must be bigger by some value. What is this value?

Extra room for finishing problems
If you use this space label your problems!