Physics 270 Exam

You may use a one-page (both sides) formula sheet for this exam. You may use a calculator---but it should not be necessary. No other written or electronic aids are allowed. The exam ends promptly at 1:50.

In problems calling for expressions as answers, give your expressions in terms of the parameters of the problem.

Show your work. To get full credit you must indicate how you obtained your answers from the physical principles studied in the course. Moreover, to receive partial credit it is essential that you make your reasoning clear. For some questions I will provide the answer (which may be useful in later sections) and ask you to explain how the answer is obtained. To get any credit for these you MUST show your reasoning---after all, you already have the correct answer.

Work the problems on the exam sheet. If you need extra space, please use the pages at the back of the exam (front and back) and label the problem number. If you do use a page in the back please indicate that you have done so.

You will be provided with a sheet of scratch paper; you can use this to check calculations before writing them down.

Name:

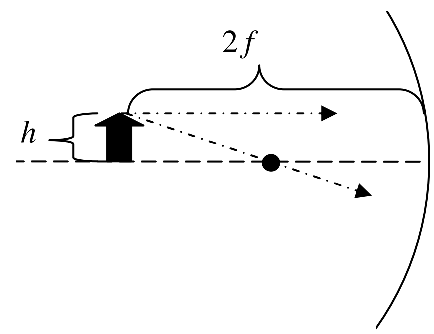
Section:

Honor Pledge:
"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination."

Signature:

1. An object of height *h* is placed in front of a convex mirror. The object is placed at distance of twice the focal length. You may assume that the height of the object is much smaller than radius of curvature of the mirror. A diagram of the set up is shown below; the dot represents the focal point, the dashed line in the center is the axis of symmetry and the two dashed arrows represent rays traveling from the top of the object---one traveling toward the focal point and the other parallel to the symmetry axis. The mirror causes an image to be formed.

   a. On the diagram to the right continue the trace of the two rays including their reflections from the mirror to find the outgoing rays. Indicate where the outgoing rays meet forming an image.



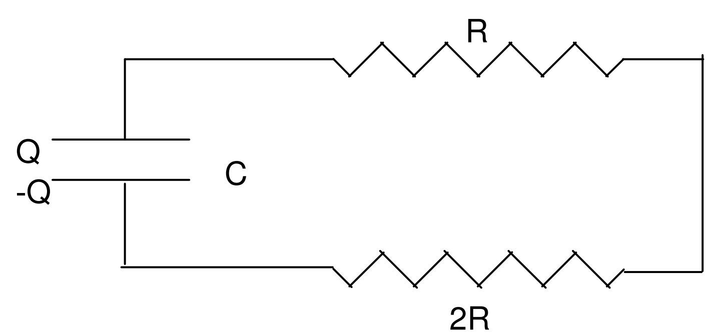
   b. Algebraically solve for the distance the image is from the mirror using the appropriate formula Express your answer in terms of *f*. Is this position consistent with your ray tracing in a?

   c. Indicate whether the following statements are true or false. (You do not need to justify your answers).

      i. The image is virtual.
      ii. The image is inverted.
      iii. The image is on the opposite side of the mirror from object.
      iv. The magnitude of the magnification is less then 2.
      v. The magnification is negative.

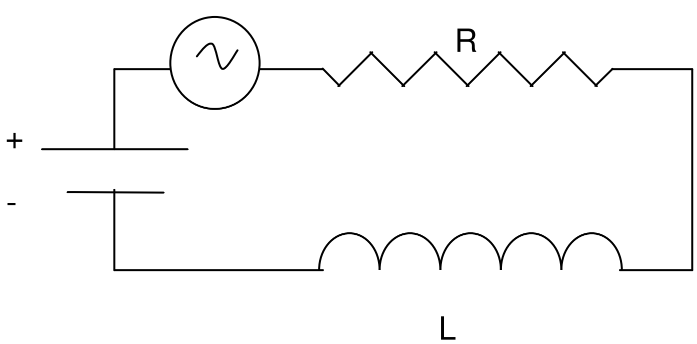
2. This problem involves differential equations describing physically important quantities for two different circuits. Parts a. and b. involve the circuit below:

a. Find a differential equation for Q, the charge on the upper plate of the capacitor.



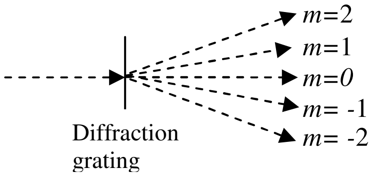
Answer part a here:

b. The charge on the capacitor for this circuit can be written in the form $Q(t) = Q_0 e^{-t/\tau}$ where Q_0 is the charge at *t=0* and τ is a constant. Find an expression for τ in terms of *R* and *C*. Hint: you can obtain this from the result in part a.



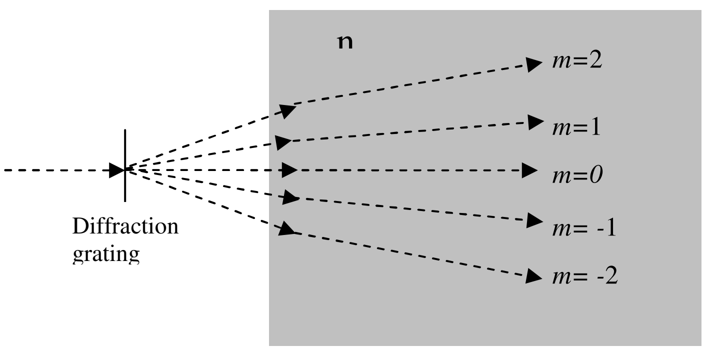
c. For the circuit to the left, find a differential equation for the current (where positive current is in the clockwise direction). The battery has a voltage of V_0 and the AC voltage source has a voltage of $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$. **You are not expected to solve this differential equation**.

3. This problem concerns both interference from a diffraction grating and effects due to a medium with an index of refraction. It useful to express results in terms of f rather than λ.



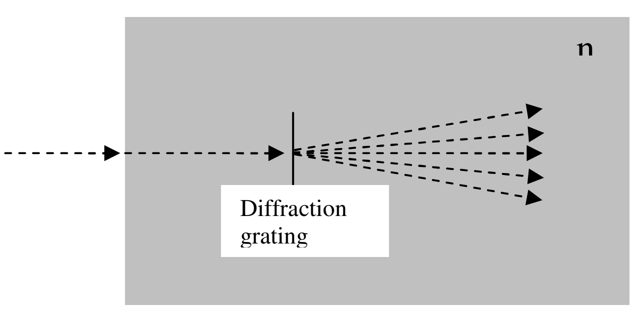
a. As a first step, consider a diffraction grating in vacuum; the slits are spaced a distance d apart. A beam of light, perpendicular to the grating and of frequency f shines on the grating as in the figure to the left. Starting from formula giving the angles for the maxima of intensity in terms of wave lengths, show that the maxima occur at angles given by

$$\sin(\theta_m) = m\frac{c}{f\ d} \text{ for } m = 0, \pm 1, \pm 2, \pm 3 \ldots.$$



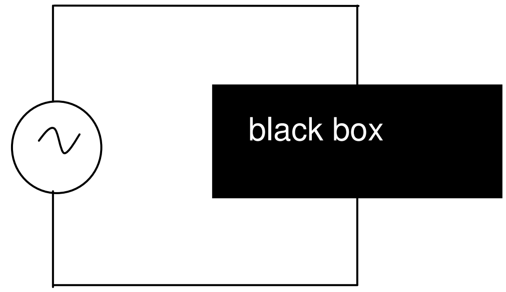
b. Suppose that some distance beyond the diffraction grating is an interface to a medium with index of refraction n. The interface is parallel to the grating. The rays leaving the grating are refracted at the surface as in the figure to the left. The dotted lines indicate rays of light; to the right of the grating these rays correspond to the maxima. Use Snell's law and the result of part a. to show that in the medium the angles for the maxima are given by

$$\sin(\theta_m^{\text{medium}}) = m\frac{c}{f\,n\,d} \text{ for}$$
$$m = 0, \pm 1, \pm 2, \pm 3 \ldots.$$



c. Consider a situation similar to part b. However instead of the grating being outside of the medium as in part b., it is inside as in the figure to the left. Derive an expression for the angles of the outgoing maxima for this situation. Are they the same as in the set up in part b?

4. Consider a “black box” which contains an arrangement of components which may include resistors, capacitors and inductors, the details of which you do not know. Two wires lead into the box an and are hooked up to an AC voltage source with $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$ as in the figure to the right: The current going into the black box is measured as a function of time and is found to be of the form: $I(t) = \frac{\mathcal{E}_0 \omega_o}{L_0}\left(A(\omega)\cos(\omega t) + B(\omega)\sin(\omega t)\right)$ where $A(\omega)$, $B(\omega)$ are measured functions, L_0 is a constant with dimensions of inductance



a. Show that the average power dissipated in the black box is $\overline{P} = \frac{\mathcal{E}_o^2 \omega_o A(\omega)}{2L_0}$. Hint: The easiest way to do this **does not require computing the phase shift.**

b. Although you cannot look into the black box, from the form of the measured current outside one can deduce whether or nor the there is a resistor in the box hooked into the circuit. Is there a resistor? How do you know? (Hint: think about the result of part a.)

c. Find an expression for the phase shift between current and voltage. Recall that if the voltage is written in the form $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$ and the current as $I(t) = I_0 \cos(\omega t - \phi)$ then the phase shift is ϕ. Express your answer in terms of $A(\omega)$ and $B(\omega)$. You may wish to use trig identities such as $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$ or $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$.

Extra room for finishing problems
If you use this space label your problems!