Homework 9:

Remember: In addition to this problem, you also have a "Mastering Physics" assignment Due April 11. Due at the **beginning** of lecture, Friday, April 11. Write up of the solution to this problem in a coherent fashion.

In this problem we will work at the details of the relativity of simultaneity for the "train/tunnel paradox" using Lorentz transformation. The length of the train in its rest frame is *L*. The tunnel is 4/5 *L* in its rest frame. The train is moving relative to the tunnel at a speed such that its γ factor is 5/3; for concreteness we will have the train moving to the left. Thus from the trains frame the tunnel is 12/25 *L* and the train does not fit entirely in the tunnel, while from the tunnel's frame the train is 3/5 *L* of a km and does fit entirely in the tunnel.

a. Show the velocity of the train in the tunnels frame is 4/5 c.
b. If one labels the tunnel's frame using unprimed space-time coordinates and the train's frame as primed space-time coordinates show that relationship between the two is given by $\begin{aligned} ct' &= \tfrac{5}{3}ct + \tfrac{4}{3}x \\ x' &= \tfrac{4}{3}ct + \tfrac{5}{3}x \end{aligned}$.

   From the tunnel's frame there are two key events, event 1:the front of the train leaving the tunnel and event 2:the back of the train of the train entering the tunnel. For convenience we will take x=0 and x=4/5 *L* to be the front end and back ends of the tunnel respectively. We will set the time t=0 to be the time when the front of the train leaves the tunnel. Thus event 1 occurs at the space-time point $(x = 0, t = 0)$ tunnel.

c. Show that the event 2 (in the tunnels frame) occurs at the space-time point $\left(x = \frac{4L}{5}, t = -\frac{L}{4c}\right)$. (You will need the fact that the train is Lorentz contracted by 5/3 to get this). Note that the train is in the tunnel as event 2 occurs before event 1.
d. Use the Lorentz transformation in b. to show that in the train's frame event 1 occurs at $(x' = 0, t' = 0)$ while event 2 occurs at $\left(x' = L, t' = \frac{13L}{20c}\right)$. Note two things about this result: x' is at L as it must be since the back of the train is at L---this shows the self-consistency of the Lorentz contraction used in c; and the fact that t'>0 indicating the front of the train leaves before the back enters.
e. Calculate the space time interval between the 2 events both in the primed and unprimed frames and verify that they are identical.