Homework 4:

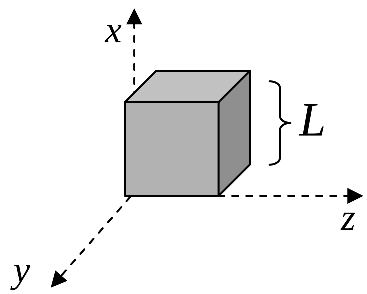
Remember: In addition to this problem, you also have a "Mastering Physics" assignment Due February 22. This assignment is due at the **beginning** of lecture, Friday, February 29. Write up of the solution to this problem in a coherent fashion.

In class we showed that the circumstances for which sinusoidal wave solved all four of Maxwell equations. There are of course also solutions which do not correspond to pure sine waves lasting for ever---after all there can be pulses of light. In fact, it can be shown that following form also solves Maxwell's equations:

$$\vec{E} = \hat{\imath} A\, f(z-ct) \quad \vec{B} = \hat{\jmath}\frac{A\, f(z-ct)}{c}$$

where A is a constant with dimensions of (force/charge) and f is an arbitrary function. This wave describes a pulse propagating in the z direction. This problem concerns the energy and energy flow of this electromagnetic wave. One can show that the Poynting vector (which describes the energy flow per unit time per unit area in the direction of propagation) defined by $\vec{S} = \dfrac{\vec{E}\times\vec{B}}{\mu_0}$ is given by $\vec{S} = c\varepsilon_0 A^2 f^2(z-ct)$. One can similarly show that the electromagnetic energy density, $u_{E-M} = \dfrac{\varepsilon_0 E^2}{2} + \dfrac{B^2}{2\mu_0}$, is given $u_{E-M} = \varepsilon_0 A^2 f^2(z-ct)$.

In this problem you will compute the electromagnetic energy inside a cube and the rate at which energy enters (or leaves) it. The cube has sides is aligned with the x, y and z axis and is placed with its corner at the origin as in the figure below:



a. Show that total electromagnetic energy inside the cube is given by $U_{EM} = \varepsilon_0 L^2 A^2 \int_0^L f^2(z-ct)\,dz$.

b. Use the Poynting vector to show the power carried by the electromagnetic wave entering the cube through the face a *z=0* (the one hidden in the figure) is given by $P_{entering} = c\varepsilon_0 L^2 A^2 f^2(-ct)$ and the power leaving the cube through the face a *z=L* (the dark one in the figure) is given by $P_{leaving} = c\varepsilon_0 L^2 A^2 f^2(L-ct)$.

c. One can use the fundamental theorem of calculus to derive the following corollary:

$$\frac{d\left(\int_0^L f^2(z-ct)\,dz\right)}{dt} = -c\left(f^2(L-ct) - f^2(-ct)\right).$$ Use this corollary to show that $\dfrac{dU_{EM}}{dt} = P_{entering} - P_{leaving}$. Explain why this makes sense on physical grounds.