Solution Quiz 3

NAME: ______________________________________	Quiz #3c: Phys270 Section 0102

1. [10 pts] Show that the displacement current inside a parallel-plate capacitor can be written as:

$$C\,\frac{dV_c}{dt}$$

where C is the capacitance and Vc is the voltage across the capacitor.

Solution:

We can use one of the Maxwells equation:

$$\oint_{\partial S} \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_S + \mu_0\varepsilon_0 \frac{\partial \Phi_{E,S}}{\partial t}$$ and rewrite it as $$\oint B.dl = \mu_0 I_s + \mu_0 I_{Disp}$$

We might say that the magnetic field B at some point outside the capacitor originates due to the effect of a real current I_s and a displacement current I_{Disp}.

Then we have

$$I_{Disp} = \varepsilon_0 \frac{\partial \Phi_{E,S}}{\partial t} = \frac{\varepsilon_0 \partial\left(\overline{E}.\overline{A}\right)}{\partial t} = \frac{\varepsilon_0 \partial \frac{V_C}{D} A}{\partial t} = \frac{\varepsilon_0 A}{D}\frac{\partial V_C}{\partial t} = C\frac{\partial V_C}{\partial t}$$

Where we have used the formula for capacitance of a parallel plate capacitor of plate separation D and plate area A (which is constant over time),

$$C = \frac{\varepsilon_0 A}{D}$$

And the relation ship between electrostatic field and electrostatic potential,

$E_x = -\frac{\partial V_x}{\partial x} = -\frac{V_C}{D}$ where x is the direction normal to the plates and the potential gradient is uniform between the plates and we have neglected the negative sign since we are dealing with magnitudes only.