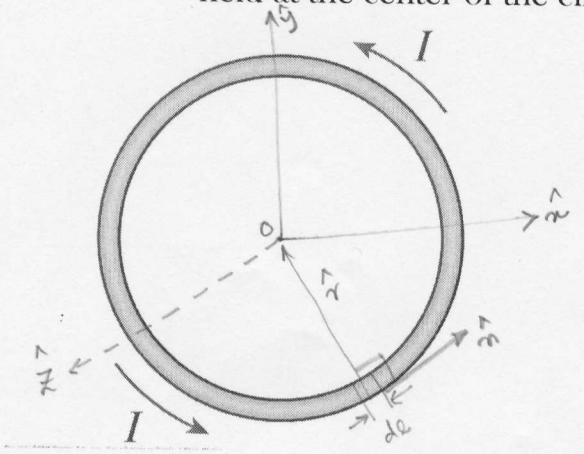
SOLUTION

NAME:	Quiz #1d: Phys270
_____	0102

1. [10 pts] A circular loop carrying current I with radius R is depicted below.
   a. [2 pts]Draw the direction of the B-field at the center of the loop on the diagram below.
   b. [8 pts] Using the Biot-Savart law, derive an expression for the magnetic field at the center of the circular loop.



$d\vec{\ell} = d\ell\,\hat{n}$

a. Using right hand thumb rule the $\vec{B}$ field is directed "outside" the paper ie. along the $\hat{z}$ axis.

b. $$d\vec{B}_{center} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\ell}\times\vec{r}}{|\vec{r}|^3}$$

we define $d\vec{\ell}$, $\vec{r}$ as shown in the picture

$\vec{r}$ and $d\vec{\ell}$ lie on the plane of the paper

$$\therefore \quad d\vec{\ell}\times\vec{r} = d\ell|\vec{r}|\,\hat{z}$$

$$\therefore \quad |\vec{r}| = R \text{ for all points on the loop}$$

$$\Rightarrow \quad d\vec{B}_{center} = \frac{\mu_0 I}{4\pi}\frac{d\ell\, R}{R^3}\hat{z}$$

$$\Rightarrow \quad d\vec{B}_{center} = \frac{\mu_0 I}{4\pi R^2}\,d\ell\,\hat{z}$$

$$\vec{B}_{center} = \frac{\mu_0 I}{4\pi R^2}\int d\ell\,\hat{z}$$

$$= \frac{\mu_0 I}{4\pi R^2}\,2\pi R\,\hat{z}$$

$$\boxed{\vec{B}_{center} = \frac{\mu_0 I}{2R}\hat{z}}$$