SOLUTION.

NAME:

Quiz #10b:
Phys270

Section 0104

1. [10 pts] Suppose that $\psi_1(x)$ and $\psi_2(x)$ are both solutions to the time-independent Schrodinger equation for the same potential energy U(x). Prove that the superposition $\psi(x) = A\,\psi_1(x) + B\,\psi_2(x)$ is also a solution to the time-independent Schrodinger equation.

Schrödinger Equation in 1 Dimension (time independent).

$$\frac{d^2}{dx^2}\psi(x) = -\frac{2m}{\hbar^2}\left[E-U(x)\right]\psi(x) \qquad \text{—— (A)}$$

Since $\psi_1(x)$ and $\psi_2(x)$ are solutions hence the satisfy

$$\frac{d^2}{dx^2}\psi_1(x) = -\frac{2m}{\hbar^2}\left[E-U(x)\right]\psi_1(x) \qquad \text{—— (1)}$$

$$\frac{d^2}{dx^2}\psi_2(x) = -\frac{2m}{\hbar^2}\left[E-U(x)\right]\psi_2(x) \qquad \text{—— (2)}$$

Lets multifily Equation (1) & (2) with constants A and B respectively and add.

$$A\frac{d^2}{dx^2}\psi_1(x) + B\frac{d^2}{dx^2}\psi_2(x) = -\frac{2m}{\hbar^2}\left[E-U(x)\right]A\psi_1(x) - \frac{2m}{\hbar^2}\left[E-U(x)\right]B\psi_2(x)$$

Since A & B are constants we can rewrite the equation as follows:

$$\frac{d^2}{dx^2}\left[A\psi_1(x) + B\psi_2(x)\right] = -\frac{2m}{\hbar^2}\left[E-U(x)\right]\left[A\psi_1(x) + B\psi_2(x)\right]$$

Substituting $\psi(x) = A\psi_1(x) + B\psi_2(x)$; where $\psi(x)$ is the linear combination of $\psi_1(x)$ & $\psi_2(x)$

we have

$$\frac{d^2}{dx^2}\psi(x) = -\frac{2m}{\hbar^2}\left[E-U(x)\right]\psi(x)$$ which is the same as (A)

∴ the superposition (linear combination) $\psi(x)$ is also a solution of the time-independent Scrödinger equation.