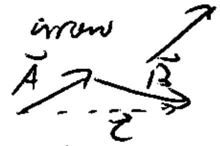
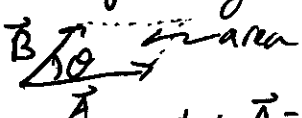
Lecture 1:

- Appendix A: Mathematical Review – Algebra, Geometry, Trig, Calculus
- Vectors: Direction + magnitude given by arrow
  - add arrows: tip-to-tale  $\vec{C} = \vec{A} + \vec{B}$
  - minus sign flips arrow head $\Rightarrow$ $\vec{C} = \vec{A} + (-\vec{B})$ or $\vec{C} = \vec{A} - \vec{B}$
  - Dot product: $\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta$ $= (|A|\cos\theta)\,|B|$  (projection of one vector onto direct of other) x length of other vector
  - Unit vectors: Magnitude is $\underline{1}$; $\hat{x}, \hat{y}, \hat{z}, \hat{r}, \ldots$
  - Cross products: $\vec{C} = \vec{A} \times \vec{B}$

    $|\vec{C}| = |A|\,|B| \sin\theta$ = area

    direction of $\vec{C}$ by R.H.R.

    ① $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

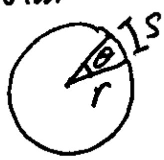
    ② $\vec{A} \times \vec{B} = 0$ $\quad \vec{A} \parallel \vec{B}$

    ③ $\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}|$ $\quad \vec{A} \perp \vec{B}$

    ④ $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times C$
  - Vector Components: $\vec{A} = \vec{A} \cdot \hat{x} + \vec{A} \cdot \hat{y}$
- Linear motion (Constant acceleration): $x,\ v = \frac{dx}{dt},\ a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$

  Constant "a" $\Rightarrow dv = a\,dt \Rightarrow v = at + v_0$

  $\frac{dx}{dt} = at + v_0 \Rightarrow x = \frac{1}{2}at^2 + v_0 t + x_0$
- Circular motion: $S = r\theta$ + r fixed

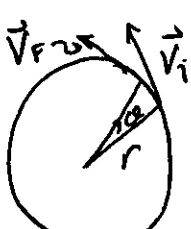
  

  $$\Rightarrow \underbrace{\frac{ds}{dt}}_{= v} = r \underbrace{\frac{d\theta}{dt}}_{= \omega} \Rightarrow v = r\omega$$

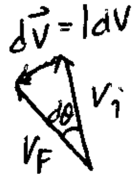
  Special Note: $\omega = 2\pi f$, $f = \frac{1}{T}$

  $$+ \quad a = r \underbrace{\frac{d^2\theta}{dt^2}}_{= \alpha}$$ ← not centripetal! tangential acceleration!

For Constant ω:



$|\vec{V}_i| = |\vec{V}_F| = |V|$ $\qquad d\vec{V} = |dV|(-\hat{r})$

For small $\theta \to d\theta$,  + $|dV| = |V|\,d\theta$

$$\therefore \underbrace{\frac{d\vec{V}}{dt}}_{= \vec{a}_c} = \frac{d}{dt}\Big(|V|\,d\theta(-\hat{r})\Big) = \underbrace{|V|}_{= r\omega} \underbrace{\frac{d\theta}{dt}}_{= \omega}(-\hat{r}) \quad \therefore \vec{a}_c = -r\omega^2\hat{r} = -\frac{V^2}{r}\hat{r}$$

- Newton's Laws:

  I. inertial Reference Frames / Law of inertia

  If the net force on a body is zero, it is possible to find a set of reference frames in which that body has no acceleration

  II. $\sum \vec{F} = m\vec{a}$

  III $\vec{F}_{12} = -\vec{F}_{21}$

- Work, Kinetic Energy, Potential Energy

$$W \equiv \int_{x_i}^{x_f} \vec{F}(\vec{x}) \cdot d\vec{x}$$

Consider 1-D where net force $\vec{F} \parallel \vec{x}$:

$$W = \int_{x_i}^{x_f} (ma)\,dx \;; \quad ma\,dx = m\frac{dv}{dt}dx = m\frac{dv}{dx}\underbrace{\frac{dx}{dt}}_{\equiv v}dx = mv\,dv$$

$$= \int_{v_i}^{v_f} mv\,dv = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}mV^2 \equiv KE$$

$\Delta U = \Delta P.E.$ = Work done by an external force

$$= \int_{x_i}^{x_f} \vec{F}_{ext}(\vec{x}) \cdot d\vec{x} = -\int_{x_i}^{x_f} \vec{F}(\vec{x}) \cdot d\vec{x}$$

fields, springs, gravity etc...

Gravity: $F = mg(\hat{y}) \Rightarrow \Delta U = mg\Delta y$

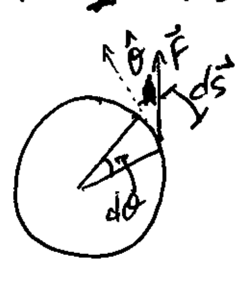
Spring: $F = -kx(\hat{x}) \Rightarrow \Delta U = -\frac{1}{2}k\Delta x^2$

- Conservation of energy & momentum

- $\vec{\tau} \equiv \vec{r} \times \vec{F}$, $\quad \sum \vec{\tau} = \Big(\sum \underbrace{F_t}_{\text{tangential Forces}}\Big) r = (ma_t)r = mr^2\alpha$

$$KE = \sum_i \frac{1}{2}m_i v_i^2 = \sum \frac{1}{2}m_i r_i^2 w^2 = \frac{1}{2}\underbrace{\left(\sum_i m_i r_i^2\right)}_{\equiv I}w^2 = \frac{1}{2}Iw^2$$

$\therefore \sum \vec{\tau} = I\alpha$



$dW = \vec{F} \cdot d\vec{s}$, $\quad d\vec{s} = r\,d\theta\,\hat{\theta}$ (fixed r!)

$\Rightarrow dW = F\sin\phi\, r\,d\theta = \tau\,d\theta \Rightarrow W = \int \tau\,d\theta$

angle between $\vec{F}$ & $\hat{r}$, $\hat{F} \cdot \hat{\theta} = \sin\phi$

& $\tau\,d\theta = I\alpha\,d\theta = I\frac{dw}{d\theta}\overbrace{\frac{d\theta}{dt}}^{w}$.

$\Rightarrow W = \int Iw\,dw = \frac{1}{2}Iw^2$

- Gravity: $\vec{F}_{12} = -G \dfrac{m_1 m_2}{r^2} \hat{r}_{12}$

Earth: $\vec{F} = \underbrace{\left(-\dfrac{G M_e}{R_e^2}\right)}_{= g} m \hat{r}$

Gravitational P.E.: $\Delta U = -\int \vec{F}_{12} \cdot dr = +G m_1 m_2 \int_{r_i}^{r_f} \dfrac{dr}{r^2}$

$\Rightarrow \Delta U = -G m_1 m_2 \left(\dfrac{1}{r_f} - \dfrac{1}{r_i}\right)$, $U \to 0$ as $r \to \infty$ ∴
Let $U \equiv 0$ @ $r_i = \infty$

$\Rightarrow U(r) = -G \dfrac{m_1 m_2}{r}$

- Simple Harmonic motion: pendulum, mass on spring

ex. $m a_x = -kx \Rightarrow \dfrac{d^2 x}{dt^2} = -\dfrac{k}{m} x \Rightarrow x = A \cos(\omega t + \phi)$, $\omega = \sqrt{\dfrac{k}{m}}$

T, period, $f \equiv \dfrac{1}{T}$, $\omega T = 2\pi \Rightarrow \omega = 2\pi f$

- Damped oscillations: $\vec{F} = -kx - b v_x$

$\Rightarrow \dfrac{d^2 x}{dt^2} = -\dfrac{k}{m} x - \dfrac{b}{m} \dfrac{dx}{dt}$; $x = A e^{-(b/2m)t} \cos(\omega t + \phi)$

where $\omega = \sqrt{\omega_0^2 - \left(\dfrac{b}{2m}\right)^2}$, $\omega_0 = \sqrt{\dfrac{k}{m}}$

- Traveling waves:

$y = A \sin(kx - \omega t + \phi)$, $k = \dfrac{2\pi}{\lambda}$, $\omega = \dfrac{2\pi}{T}$

Surfing on wave (constant phase) $\Rightarrow kx - \omega t + \phi = \text{constant}$

$\Rightarrow \dfrac{d}{dt}(kx - \omega t + \phi) = 0 \Rightarrow \dfrac{dx}{dt} = \underbrace{V_\phi}_{\text{phase velocity}} = \dfrac{\omega}{k}$

or $V_\phi = \dfrac{2\pi f}{2\pi/\lambda} = \lambda f$

- Chapter 21 extremely important for optics & quantum mechanics
  - Standing waves
  - interference
  - phase differences, path length

Will Review later

• Coulomb's law: $\vec{F} = k_e \dfrac{q_1 q_0}{r^2}\hat{r}$ $\quad q_1 \bullet \overset{r}{\frown\frown} \bullet q_0$

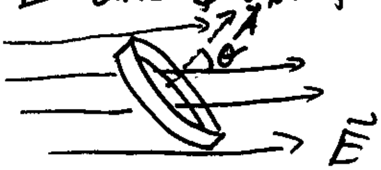
• Electric Field: $\vec{E} \equiv \dfrac{\vec{F}_e}{q_0}$ & Since $\vec{F}_{tot} = \sum_i \vec{F}_{i0} \Rightarrow \vec{E}_{tot} = \sum_i \vec{E}_i$

$$\vec{E}_{tot} = \sum_i k_e \frac{dq_i}{r^2}\hat{r} = \int k_e \frac{dq}{r^2}\hat{r}$$

Examples: line charges, hoops, disks, infinite sheets (ρ, σ, λ)

Electric field lines: ① tangent to $\vec{E}$ ② density of field lines $\propto |E|$

• Electric flux: for constant & uniform $\vec{E}$ & $\vec{A}$,



$\Phi_E = \vec{E}\cdot\vec{A} \propto$ Net # of field lines through surface $\vec{A}$

$= |E||A|\cos\theta$

• Gauss's law: $\Phi_E = \oint \vec{E}\cdot d\vec{a} = \dfrac{Q_{inc}}{\varepsilon_0}$; ex's spheres, coax's, lines, ∞-ite sheets, Conductors

• Conductors in electrostatic equilibrium:
① $\vec{E}=0$ inside ② Charge $\exists$ only on surface ③ $\vec{E} \perp$ surface
④ V constant throughout conductor

• Electric Potential:

$$\Delta U = W_{ext} = -\int_{x_i}^{x_f} \vec{F}_e \cdot d\vec{x}, \quad \text{electric } \vec{F}_e = q_0\vec{E}$$

$$= -q_0 \int_{x_i}^{x_f} \vec{E}\cdot d\vec{x} \quad ;$$

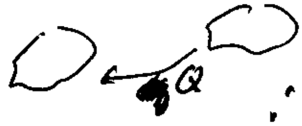
$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_{x_i}^{x_f} \vec{E}\cdot d\vec{x}$$

① Constant & uniform $\vec{E}$ where $d\vec{x} \parallel \vec{E} \Rightarrow \Delta V = -E\Delta x$

② for a pt. charge: $\Delta V = -\int_{r_i}^{r_f} k_e \dfrac{q}{r^2}\,dr \Rightarrow V = \dfrac{k_e q}{r}$ if $V_i @ r_i$ chosen $\ni V_i = 0$

③ for a bunch of charges: $V = \sum_i k_e \dfrac{q_i}{r_i} \longrightarrow k_e \int \dfrac{dq}{r}$

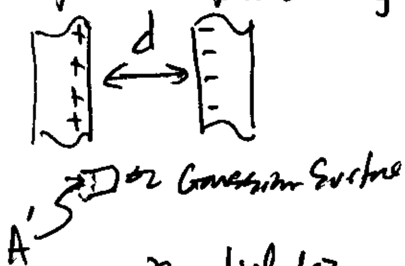
④ Equipotential surfaces: always $\perp$ to $\vec{E}$ (& $\vec{F}_e$!),

- Capacitance:  $\Delta V \propto |E|$, & $|E| \propto Q$

$\therefore \Delta V \propto Q$

$$C \equiv \left|\frac{Q}{\Delta V}\right|$$

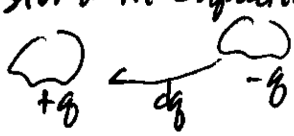
parallel plates: $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{inc}}{\varepsilon_0} \Rightarrow E A' = \frac{\frac{Q}{A} A'}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$



& $\Delta V = -Ed = \frac{Qd}{A\varepsilon_0} \Rightarrow C = \frac{A\varepsilon_0}{d}$

with dielectric, $\vec{E}_0 \to \vec{E}_0/K$

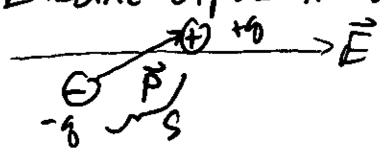
Energy stored in capacitor: $du = \frac{q}{C}\,dq \Rightarrow \Delta u = \frac{1}{2}\frac{Q^2}{C}$, & $\Delta V = \frac{Q}{C} = Ed$ for ∥ plate



$\Rightarrow \Delta u = \frac{1}{2}\frac{1}{C}(EdC)^2 = \frac{1}{2} dC\, E^2 = \left(\frac{1}{2}A\varepsilon_0\right)E^2$

$\Rightarrow \Delta u \propto E^2$, energy density

- Electric dipole in a uniform Electric field



$\vec{\tau} = \vec{p}\times\vec{E}$, $\vec{p} = qs$ from "-" to "+"

$u = -\vec{p}\cdot\vec{E}$

- Ohm's law & currents:

$J \equiv \frac{\Delta Q}{A_\perp \Delta t}$, current density. "Amount of charge ΔQ passing through a cross-sectional area $A_\perp$ per unit time Δt"

$I = \vec{J}\cdot\vec{A} = \frac{dQ}{dt}$

Drude model / RTA (ohmic) $\Rightarrow \vec{J} = nq\vec{V}_D = \underbrace{\frac{ne^2\tau}{m}}_{\equiv\,\sigma\text{, conductivity}}\vec{E}$ ($\vec{V}_D$: drift velocity, τ: scattering time)

Ohm's law: $\Delta V = IR$; $R = \rho\frac{l}{A}$; $\rho = \frac{1}{\sigma}$, resistivity

$P = I(\Delta V)$

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- DC Circuits:

|| Capacitors: $C_{eq} = C_1 + C_2$

Series " : $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

|| Resistors: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

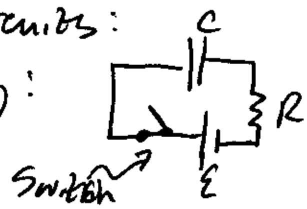
Series Resistors: $R_{eq} = R_1 + R_2$

Kirchoff rules: ① $I_{in} = I_{out}$ (@ junctions) i.e. no charge build up

② $\sum_{\text{Loop}} \Delta V_i = 0$ Conservation of energy

- RC Circuits:

① Charging:



Kirchoff loop: $\varepsilon - \frac{q}{C} - \frac{dq}{dt} R = 0$

$\Rightarrow q(t) = Q\left(1 - e^{-t/RC}\right), \; I = \frac{\varepsilon}{R} e^{-t/RC}$

② Discharging

$-\frac{q}{C} - \frac{dq}{dt} R = 0$

$\Rightarrow q(t) = Q e^{-t/RC}, \; I = \frac{Q}{RC} e^{-t/RC}$