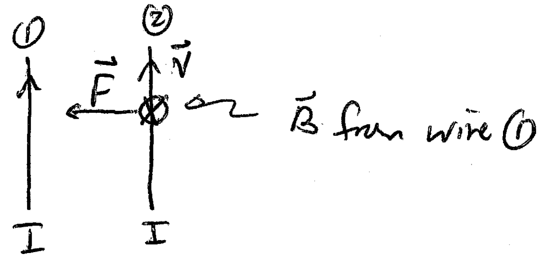
- Properties of permanent Magnets: ⑤ This is what a compass is!
  ① opp. poles attract ② like poles repel ③ magnets do not cause forces on stationary charges ④ Cannot isolate poles: Break a magnet & you get 2! ⑤ magnets can pick up ferrous materials ⑥ Magnets (Compass) pt in direction of $\vec{B}$
- Connection between electricity & magnetism:
  - 1819, Oersted during a classroom demonstration: current causes compass to ~~jump~~ move
  - Compasses ~~??~~ align w/ magnetic field
    ⇒ North pole pts in direction of applied B-field
    (Don't worry about How for the moment!)
  - Picture of current carrying wire w/ ~~metal~~ iron filings: shows shape of $\vec{B}$-field, tangent to circles around wire $\boxed{\text{Current (moving charge) creates a magnetic field!}}$
  - Like Electric field lines are drawn tangential to $\vec{E}$-field, Magnetic field lines are drawn tangential to $\vec{B}$; RHR - thumb along current

$\boxed{\text{Moving charges (currents) creates a magnetic field}}$

  - Consider 2 wires with ∥ currents:
    - wire 2 is attracted to ①
    - ~~Note, a stationary charge q feels no force~~
    - Note: wire is neutral so can not be an electrostatic force!
    - must be moving charges in $\vec{B}$-field feel a force!



looks like cross product: phenomenologically found:

$$\vec{F}_B = q\,\vec{v}\times\vec{B}$$

Charge q moving @ velocity $\vec{v}$ in magnetic field $\vec{B}$ feels a force $\vec{F}$. This defines $\vec{B}$!!

$$\boxed{B \sim \frac{F}{qv} = \frac{N}{C\,m/s} = \frac{N}{A\cdot m} = T}$$

$\boxed{\text{Moving charges in magnetic field feels a force}}$

① $\vec{F}_B \perp$ to $\vec{v}$ & $\vec{B}$

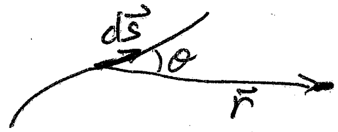
② Must be a moving charge to feel a force

③ Corollary to ①: $\vec{F}_B$ does no work:

$$W = \underbrace{\int \vec{F}\cdot d\vec{x}}_{=0} = \int \vec{F}\cdot\vec{v}\,dt \quad \& \quad F \perp v$$

Skip until later

Biot - Savart law: experimentall found!



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

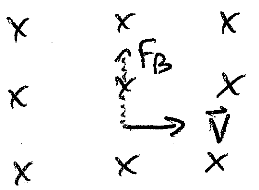
$d\vec{s}$ is along wire in direction of current

$d\vec{B} \perp$ to $d\vec{s}$ & $\hat{r}$

For cts wire: $\vec{B} = \frac{\mu_0}{4\pi} I \int \frac{d\vec{s} \times \hat{r}}{r^2}$

μ_0 = permeability of free space $= 4\pi \cdot 10^{-7}\ Tm/A$

• Assume a B-field exists in space & is constant & uniform:



$F_B = q\vec{v} \times \vec{B}$, $V \perp B$, & assume $q > 0$

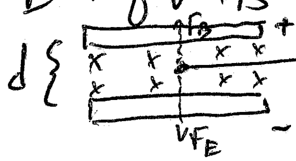
$F_B = qVB = m a_c = m\frac{V^2}{r} \Rightarrow \frac{V}{r} = \frac{q}{m} B$

or $\omega = \frac{q}{m} B$, cyclotron frequency

~~Magnetic force on current carrying wire~~:

• Lorentz Force: $F = q\vec{E} + q\vec{V} \times \vec{B}$ assume $q > 0$

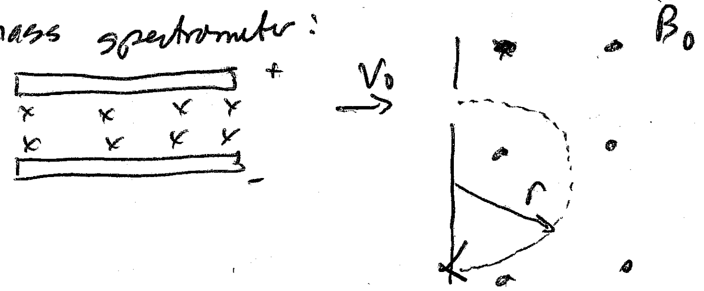
① velocity selector:



$F = 0 = qE + qVB$

$\Rightarrow V = \frac{E}{B}$; $\Delta V = E \cdot d$

② mass spectrometer:



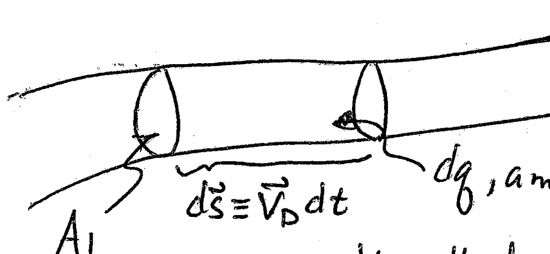
$V_0 = E/B$

$F_B = qVB_0 = m\frac{V_0^2}{r}$

$\Rightarrow \frac{m}{q} = \frac{rB_0}{V_0} = \boxed{\frac{rBB_0}{E}}$

• Magnetic force on a current-carrying wire:

$dF_B = dq\, \vec{V}_D \times \vec{B} = \frac{dq}{dt}(dt\vec{V}_D) \times \vec{B}$; $\vec{V}_D = \frac{d\vec{s}}{dt}$

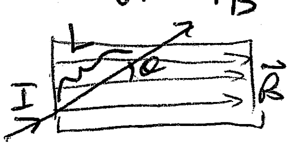


dq, amt of charge in volume, $= n(A_\perp \overbrace{V_D dt}^{ds}) q_0$ ← charge per carrier

in time dt, all charge dq in volume will leave ∴ $I = \frac{dq}{dt}$

$d\vec{F}_B = I\, d\vec{s} \times \vec{B}$ or $\vec{F}_B = \int_{s_1}^{s_2} I\, d\vec{s} \times \vec{B}$

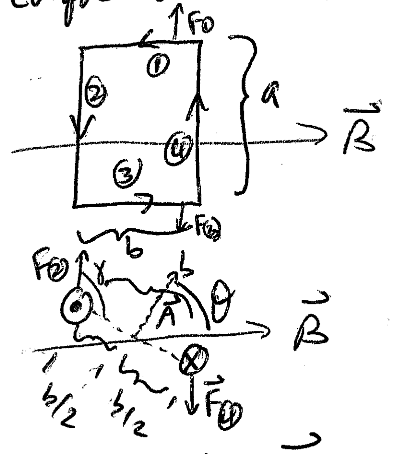
For a straight wire: Uniform field



$F_B = I\int_0^L ds\, B\sin\theta = ILB\sin\theta$

$\Rightarrow \boxed{F_B = I\vec{L} \times \vec{B}}$

- Torque on a Current loop in a uniform magnetic field



$\vec F = I\vec L\times\vec B$

$F_1 = -F_3$ & does not torque on loop

F_2 & F_4 Torque on loop: $I\vec L \perp \vec B$, $L = a$, $r = b/2$

$\tau = \vec r\times\vec F = rF_2\sin\gamma + rF_4\sin\gamma$

RHR defines $\vec A$

$\gamma + \theta = \pi$ & $\sin\gamma = \sin(\pi - \theta) = \sin\theta$

$\Rightarrow \tau = \frac{b}{2}(IaB)\sin\theta + \frac{b}{2}(IaB)\sin\theta = I(ab)\sin\theta\, B$

$= I\vec A\times\vec B$, Let $\vec\mu \equiv I\vec A$ "magnetic dipole moment"

$\Rightarrow \boxed{\vec\tau = \vec\mu\times\vec B}$ Similar to electric dipole, $\vec\tau = \vec p\times\vec E$, $p = 2aq$

$$U = \int_{\theta_i}^{\theta_f} \tau_B\, d\theta = \int_{\theta_i}^{\theta_f}(\mu B\sin\theta)\,d\theta = -\mu B\cos\theta\Big|_{\theta_i}^{\theta_f}$$

$\tau_B \parallel$ to $\hat\theta$

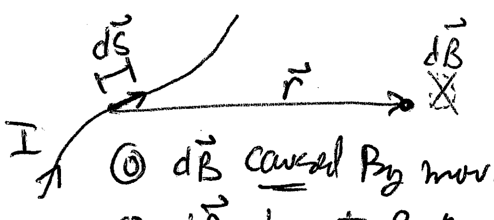
Let $U=0$ when $\theta = 90°$, & Let $\theta_i = 90°$, $\theta_f \to \theta$ ~~measured away from~~

$\therefore U = -\vec\mu\cdot\vec B$ Similar to electric dipole, $U = \vec p\cdot\vec E$

- $\vec F = q\vec v\times\vec B$ or $d\vec F_B = (I\,d\vec s)\times\vec B$ defines $\vec B$

By measuring the Force on current element $(I\,d\vec s)$, we can Find magnitude & direction of $\vec B$.
By doing this many times, one finds the B-field produced by a current element:

Biot-Savart Law: $d\vec B = \frac{\mu_0}{4\pi}\frac{(I\,d\vec s)\times\hat r}{r^2}$, magnetic field produced by current element $(I\,d\vec s)$, where $d\vec s$ in direction of I, a distance r away in direction of $\hat r$
(phenomenological)



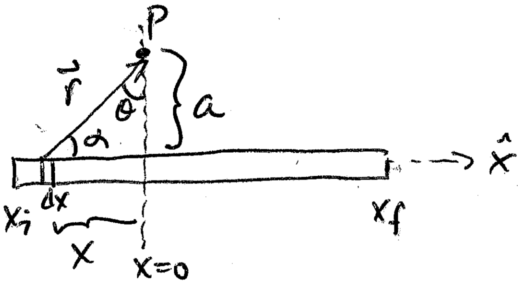
μ_0 = magnetic permiability of free space $= 4\pi\cdot 10^{-7}\ Tm/A$

0. $d\vec B$ caused By moving charge $(I\,d\vec s)$
1. $d\vec B \perp$ to Both the current & $\hat r$
2. looks kinda like E field produced by a bit of charge dq: $d\vec E = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat r$
that is, magnitude $\propto \frac{1}{r^2}$

For Cts wire: $\vec B(\vec r) = \frac{\mu_0}{4\pi} I \int \frac{d\vec s\times\hat r}{r^2}$

Biot Savart law examples:

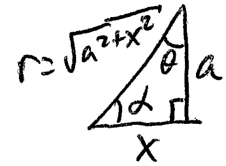
- Serway ex 30.1 : Finite straight wire w/ I constant. Find B @ Pt. P



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{(I\,d\vec{s})\times\hat{r}}{r^2}$$

$$d\vec{s} = dx\,\hat{x} \quad ; \quad \hat{x}\times\hat{r} = |\hat{x}||\hat{r}|\sin\alpha\,\hat{z} = \cos\theta\,\hat{z}$$

$$\cos\theta = \frac{a}{\sqrt{a^2+x^2}} \quad , \quad r = \sqrt{a^2+x^2}$$



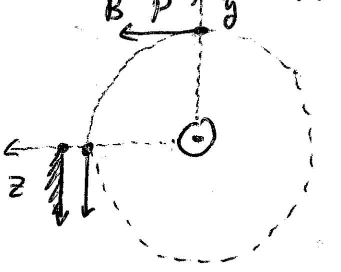
$$\therefore\ d\vec{B} = \frac{\mu_0}{4\pi} I \frac{dx\ \hat{x}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi} I \frac{dx\ a}{(a^2+x^2)^{3/2}}\hat{z}$$

Appendix A: $\displaystyle\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$

$$\therefore\ \vec{B} = \int_{x_i}^{x_f}\left(\frac{\mu_0}{4\pi} I a\right)\frac{dx}{(a^2+x^2)^{3/2}}\hat{z} = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\frac{x_f}{\sqrt{x_f^2+a^2}} - \frac{x_i}{\sqrt{x_i^2+a^2}}\right]$$

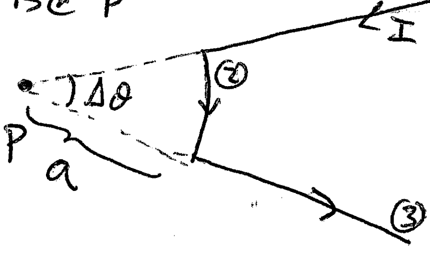
Let $x_f = L$, $x_i = -L$, & let $L \to \infty$ $(L \gg a)$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{I}{a}\frac{2L}{\sqrt{L^2+a^2}}\hat{z} = \frac{\mu_0}{4\pi}\frac{I}{a}\cdot 2\cancel{L}\frac{1}{\cancel{L}\sqrt{1+\underbrace{(a/L)^2}_{\to 0 \ \lim L\to\infty}}}\hat{z} = \boxed{\frac{\mu_0}{2\pi}\frac{I}{a}\hat{z}\ \infty\text{-ite wire}}$$



$\vec{B}$
① tangent to circles around wire $\longrightarrow$ B field lines are circles
② Same for all x (B does not depend on x)
③ $|B|$ constant @ constant radius
④ RHR! Thumb along current, fingers curl in direction of $\vec{B}$

- Serway ex 30.2 :
Find B @ P



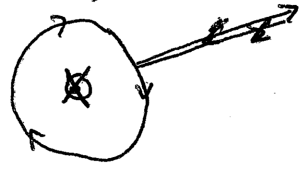
$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{s}\times\hat{r}}{r^2} \quad ; \quad \text{For ① \& ③ } d\vec{s}\parallel\hat{r} \Rightarrow d\vec{s}\times\hat{r} = 0$$

For ② $d\vec{s}\perp\hat{r} \Rightarrow d\vec{s}\times\hat{r} = ds$ into page
& $ds = a\,d\theta$

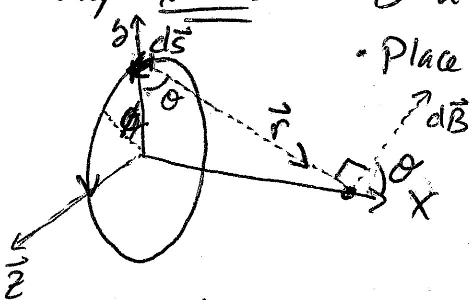
$$|B| = \frac{\mu_0}{4\pi} I \int_0^{\Delta\theta}\frac{a\,d\theta}{a^2} = \frac{\mu_0}{4\pi}\frac{I}{a}\Delta\theta$$

For a full circle (P @ center of circle!), $\Delta\theta \to 2\pi$

$$\Rightarrow |B| = \frac{\mu_0 I}{2a} \text{ into page}$$



Serway ex 30.3: $\vec{B}$ along axis of a circular current loop:



• Place loop in y-z plane centered on x-axis; radius of loop $= a$

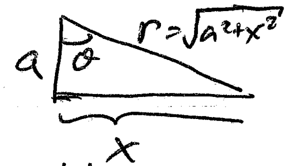
$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{I\,d\vec{s}\times\hat{r}}{r^2}$$

$d\vec{s}\times\hat{r}$ is $\perp$ to both $d\vec{s}$ & $\vec{r}$. Also $d\vec{s}\perp\hat{r} \Rightarrow |d\vec{s}\times\hat{r}| = ds = a\,d\phi$

When integrate around ~~surface~~ (loop), all y & z components will cancel.

[Show MIT widgets]

$$dB_x = d\vec{B}\cdot\vec{x} = |dB|\cos\theta\,,$$



$$\cos\theta = \frac{a}{\sqrt{a^2+x^2}}$$

$$dB_x = \frac{\mu_0}{4\pi}\,I\,\frac{|d\vec{s}\times\hat{r}|}{r^2}\cos\theta = \frac{\mu_0}{4\pi}\,I\,\frac{a\,d\phi}{(a^2+x^2)}\,\frac{a}{\sqrt{a^2+x^2}}$$

$$\therefore\ B_x = \int_0^{2\pi}\left[\frac{\mu_0}{4\pi}\,I\,\frac{a^2}{(a^2+x^2)^{3/2}}\right]d\phi = \frac{\mu_0}{2}\,\frac{Ia^2}{(x^2+a^2)^{3/2}}$$

$$\lim_{x\to 0} \Rightarrow B_x = \frac{\mu_0 I}{2a}\ \checkmark$$

Let $x \gg a$, $\quad B_x = \dfrac{\mu_0}{2}\,Ia^2\,\dfrac{1}{x^3\underbrace{\left(1+\frac{a^2}{x^2}\right)^{3/2}}_{\sim 1}}$

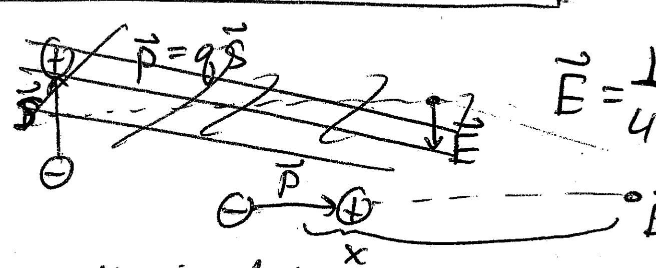
$$\Rightarrow \boxed{B_x = \frac{\mu_0}{2}\,Ia^2\,\frac{1}{x^3}}\,,\ \text{in } \hat{x} \text{ direction} = \hat{A} \text{ direction!}$$

$$\Rightarrow B_x = \frac{\mu_0}{4\pi}\,\frac{2(\pi a^2)I}{x^3}\ \text{ in } \hat{A} \text{ direction, } A = \pi a^2$$

Let $\vec{\mu} \equiv I\vec{A}$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0}{4\pi}\,\frac{2\vec{\mu}}{x^3}\quad \text{on axis of current loop}}$$

looks like electric dipole:



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\,\overset{\text{on-axis}}{\frac{2\vec{p}}{x^3}}$$

$\therefore$ ~~has~~ current loops are "magnetic dipoles"