Classical mechanics:

x(t), y(t), z(t) specifies the system completely. Or,

x(to), y(to),z(to), px(to), py(to),pz(to) (initial position and momentum) together with Force specifies the system completely.

Essentially, the force can be viewed as an operator which propagates the time evolution of the system:

$$\vec{F} = m\vec{a} = m\frac{\partial^2}{\partial t^2}\vec{x} = -\nabla V(\vec{x})$$

or

$$\frac{\partial}{\partial t}\vec{p} = -\nabla V(\vec{x})$$

U(x,y,z)

$$E(x, y, z, \dot{x}, \dot{y}, \dot{z}) \rightarrow H(x, y, z, p_x, p_y, p_z) = \frac{1}{2m}(p_x{}^2 + p_y{}^2 + p_z{}^2) + \frac{q\Phi_0}{d} z \tag{1.13}$$

The energy, written in this manner, as a function of coordinates and momenta is called the *Hamiltonian*, H. One speaks of p_x as being the momentum *conjugate* to x; p_y is the momentum conjugate to y; and so on.

The equations of motion (i.e., the equations that replace Newton's second law) in Hamiltonian theory are (for a point particle moving in three-dimensional space)

$$\begin{aligned} \frac{\partial H}{\partial x} &= -\dot{p}_x & \frac{\partial H}{\partial p_x} &= \dot{x} \\ \frac{\partial H}{\partial y} &= -\dot{p}_y & \frac{\partial H}{\partial p_y} &= \dot{y} \\ \frac{\partial H}{\partial z} &= -\dot{p}_z & \frac{\partial H}{\partial p_z} &= \dot{z} \end{aligned} \tag{1.14}$$

1.3 THE STATE OF A SYSTEM

To know the values of the generalized coordinates of a system at a given instant is to know the location and orientation of the system at that instant. In classical physics we can ask for more information about the system at any given instant. We may ask for its motion as well. The location, orientation, and motion of the system at a given instant specify the state of the system at that instant. For a point particle in 3-space, the classical state Γ is given by the six quantities (Fig. 1.14)

$$\Gamma = (x, y, z, \dot{x}, \dot{y}, \dot{z}) \tag{1.36}$$

In terms of momenta,

$$\Gamma = (x, y, z, p_x, p_y, p_z) \tag{1.37}$$

In quantum mechanics, one can not concurrently specify definite quantities of some variables (like x and px for example) to characterize the state of the system (due to Heisenberg uncertainty).

What variables can one concurrently specify which give the maximum information of the state of a system ("good" quantum numbers)?

- Consider free particle in 1D: x, and E
- Consider Hydrogen atom: Lz, L, and E

Can specify other pairs of quantities in other 'representations'.

In classical mechanics, Newton's laws of motion determines how the system changes in time.

In quantum mechanics, Schrodinger's equation determines how the system changes in time.

Postulates of Quantum Mechanics from Liboff, Introduction to Quantum Mechanics

Postulate I

This postulate states the following: To any self-consistently and well-defined observable in physics (call it A), such as linear momentum, energy, mass, angular momentum, or number of particles, there corresponds an operator (call it $\hat{A}$) such that measurement of A yields values (call these measured values a) which are eigenvalues of $\hat{A}$. That is, the values, a, are those values for which the equation

$$\hat{A}\varphi = a\varphi \qquad (3.1)$$

an eigenvalue equation

has a solution φ. The function φ is called the *eigenfunction* of $\hat{A}$ corresponding to the eigenvalue a.

TABLE 3.1 Examples of operators

$\hat{D} = \partial/\partial x$	$\hat{D}\varphi(x) = \partial\varphi(x)/\partial x$
$\hat{\Delta} = -\partial^2/\partial x^2 = -\hat{D}^2$	$\hat{\Delta}\varphi(x) = -\partial^2\varphi(x)/\partial x^2$
$\hat{M} = \partial^2/\partial x\,\partial y$	$\hat{M}\varphi(x, y) = \partial^2\varphi(x, y)/\partial x\,\partial y$
$\hat{I}$ = operation that leaves φ unchanged	$\hat{I}\varphi = \varphi$
$\hat{Q} = \int_0^1 dx'$	$\hat{Q}\varphi(x) = \int_0^1 dx'\varphi(x')$
$\hat{F}$ = multiplication by $F(x)$	$\hat{F}\varphi(x) = F(x)\varphi(x)$
$\hat{B}$ = division by the number 3	$\hat{B}\varphi(x) = \frac{1}{3}\varphi(x)$
$\hat{\Theta}$ = operator that annihilates φ	$\hat{\Theta}\varphi = 0$
$\hat{P}$ = operator that changes φ to a specific polynomial of φ	$\hat{P}\varphi = \varphi^3 - 3\varphi^2 - 4$
$\hat{G}$ = operator that changes φ to the number 8	$\hat{G}\varphi = 8$

$$\hat{p}_x = -i\hbar\frac{\partial}{\partial x} \tag{3.3}$$

The eigenvalue equation for this operator is

$$-i\hbar\frac{\partial}{\partial x}\varphi = p_x\varphi \tag{3.4}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\mathbf{r}) = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})$$

time-independent Schrödinger equation. $\hat{H}\varphi(\mathbf{r}) = E\varphi(\mathbf{r})$

Free Particle: $\hat{H} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$ $\quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi = E\varphi$

$$E = \frac{\hbar^2 k^2}{2m} \qquad p = \hbar k$$

Postulate II

The second postulate[1] of quantum mechanics is: measurement of the observable A that yields the value a leaves the system in the state φ_a, where φ_a is the eigenfunction of $\hat{A}$ that corresponds to the eigenvalue a.

As an example, suppose that a free particle is moving in one dimension. We do not know which state the particle is in. At a given instant we measure the particle's momentum and find the value $p = \hbar k$ (with k a specific value, say $1.3 \times 10^{10}\ \mathrm{cm}^{-1}$). This measurement[2] leaves the particle in the state φ_k, so immediate subsequent measurement of p is certain to yield $\hbar k$.

Postulate III

The third postulate of quantum mechanics establishes the existence of the state function and its relevance to the properties of a system: The state of a system at any instant of time may be represented by a state or wave function ψ which is continuous and differentiable. All information regarding the state of the system is contained in the wavefunction. Specifically, if a system is in the state $\psi(\mathbf{r}, t)$, the average of any physical observable C relevant to that system at time t is

$$\langle C \rangle = \int \psi^* \hat{C} \psi \, d\mathbf{r} \tag{3.32}$$

(The differential of volume is written $d\mathbf{r}$.) The average, $\langle C \rangle$, is called the *expectation value* of C.

Postulate IV

The fourth postulate of quantum mechanics specifies the time development of the state function $\psi(\mathbf{r}, t)$: the state function for a system (e.g., a single particle) develops in time according to the equation

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \hat{H}\psi(\mathbf{r}, t) \tag{3.45}$$

This equation is called the *time-dependent Schrödinger* equation.[1] The operator $\hat{H}$ is the Hamiltonian operator. For a single particle of mass m, in a potential field $V(\mathbf{r})$, it is given by (3.12). If $\hat{H}$ is assumed to be independent of time, we may write

The time-dependent Schrödinger equation permits solution of the initial-value problem: given the initial value of the state function $\psi(\mathbf{r}, 0)$, determine $\psi(\mathbf{r}, t)$. We will

The following are from Cohen-Tannoudji, Quantum Mechanics:

First Postulate: At a fixed time t_0, the state of a physical system is defined by specifying a ket $| \psi(t_0) \rangle$ belonging to the state space $\mathscr{E}$.

It is important to note that, since $\mathscr{E}$ is a vector space, this first postulate implies a superposition principle : a linear combination of state vectors is a state vector. We shall discuss this fundamental point and its relations to the other postulates in §E.

Second Postulate: Every measurable physical quantity $\mathscr{A}$ is described by an operator A acting in $\mathscr{E}$; this operator is an observable.

(*ii*) Unlike classical mechanics (*cf.* § A), quantum mechanics describes in a fundamentally different manner the state of a system and the associated physical quantities : a state is represented by a vector, a physical quantity by an operator.

Third Postulate: The only possible result of the measurement of a physical quantity $\mathscr{A}$ is one of the eigenvalues of the corresponding observable A.

Fourth Postulate (case of a discrete non-degenerate spectrum): When the physical quantity $\mathscr{A}$ is measured on a system in the *normalized* state $|\psi\rangle$, the probability $\mathscr{P}(a_n)$ of obtaining the *non-degenerate* eigenvalue a_n of the corresponding observable A is:

$$\mathscr{P}(a_n) = |\langle u_n | \psi \rangle|^2$$

where $| u_n \rangle$ is the normalized eigenvector of A associated with the eigenvalue a_n.

Fourth Postulate (case of a continuous non-degenerate spectrum): When the physical quantity $\mathscr{A}$ is measured on a system in the *normalized* state $|\psi\rangle$, the probability $\mathrm{d}\mathscr{P}(\alpha)$ of obtaining a result included between α and $\alpha + \mathrm{d}\alpha$ is equal to:

$$\mathrm{d}\mathscr{P}(\alpha) = |\langle v_\alpha | \psi \rangle|^2 \, \mathrm{d}\alpha$$

where $|v_\alpha\rangle$ is the eigenvector corresponding to the eigenvalue α of the observable A associated with $\mathscr{A}$.

Fifth Postulate: If the measurement of the physical quantity $\mathscr{A}$ on the system in the state $|\psi\rangle$ gives the result a_n, the state of the system immediately after the measurement is the normalized projection, $\dfrac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}$, of $|\psi\rangle$ onto the eigensubspace associated with a_n.

Sixth Postulate: The time evolution of the state vector $|\psi(t)\rangle$ is governed by the Schrödinger equation:

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

where $H(t)$ is the observable associated with the total energy of the system.

Pauli Exclusion principle for Fermions:

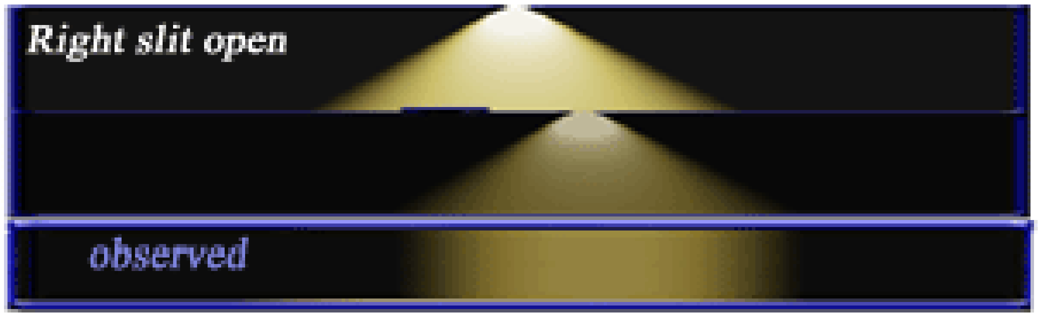
When a system includes several identical particles, only certain kets of its state space can describe its physical states. Physical kets are, depending on the nature of the identical particles, either completely symmetric or completely antisymmetric with respect to permutation of these particles. Those particles for which the physical kets are symmetric are called *bosons*, and those for which they are antisymmetric, *fermions*.

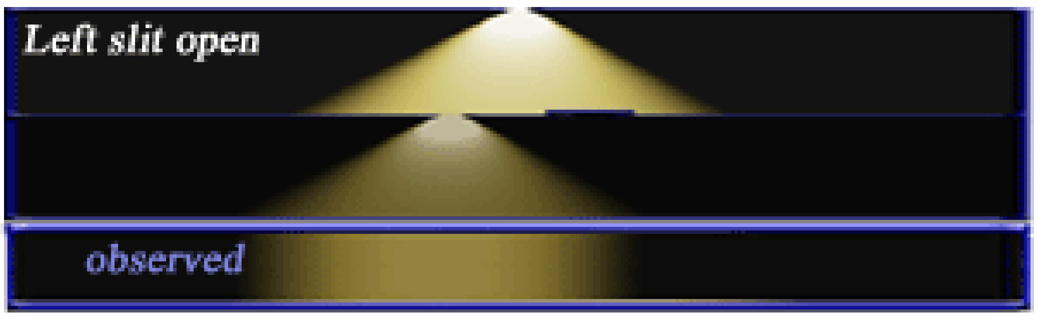
Postulates did not state anything regarding what a measurement actually is!

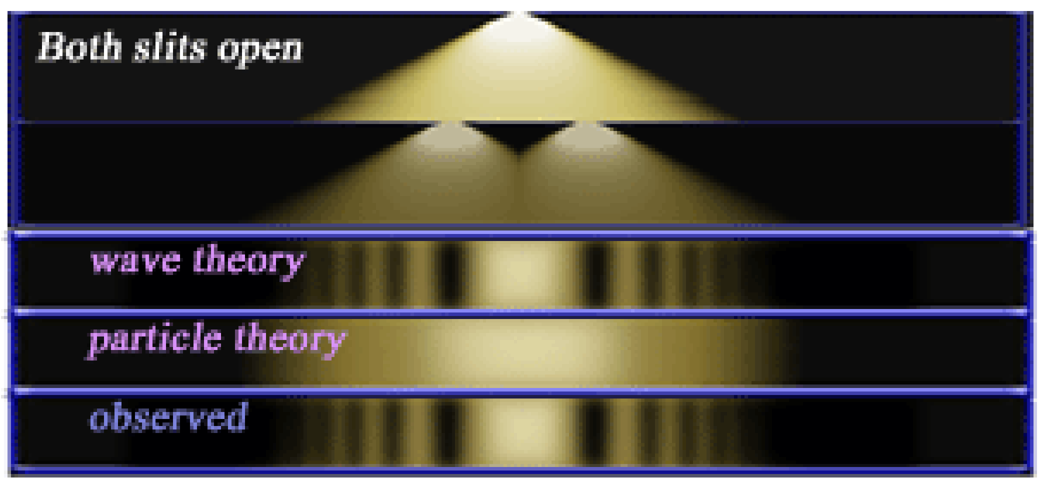
Postulates only state that there is a corresponding operator associated with a measurement, and that the wavefunction ‘collapses’ into a specific eigenvector (of this operator) after the measurement is made.

What is a measurement? What do people mean when they state the ‘collapse of the wavefunction’?

Collapse of the wavefunction: Double Slit example







Consider single electron through double slit:

Immediately before measurement (before striking the detector), ψspread out over entire screen --- x uncertain.

Immediately after measurement, ψ no longer spread out --- x known.

Wave function collapsed into a definite position.

Interpretations of QM:

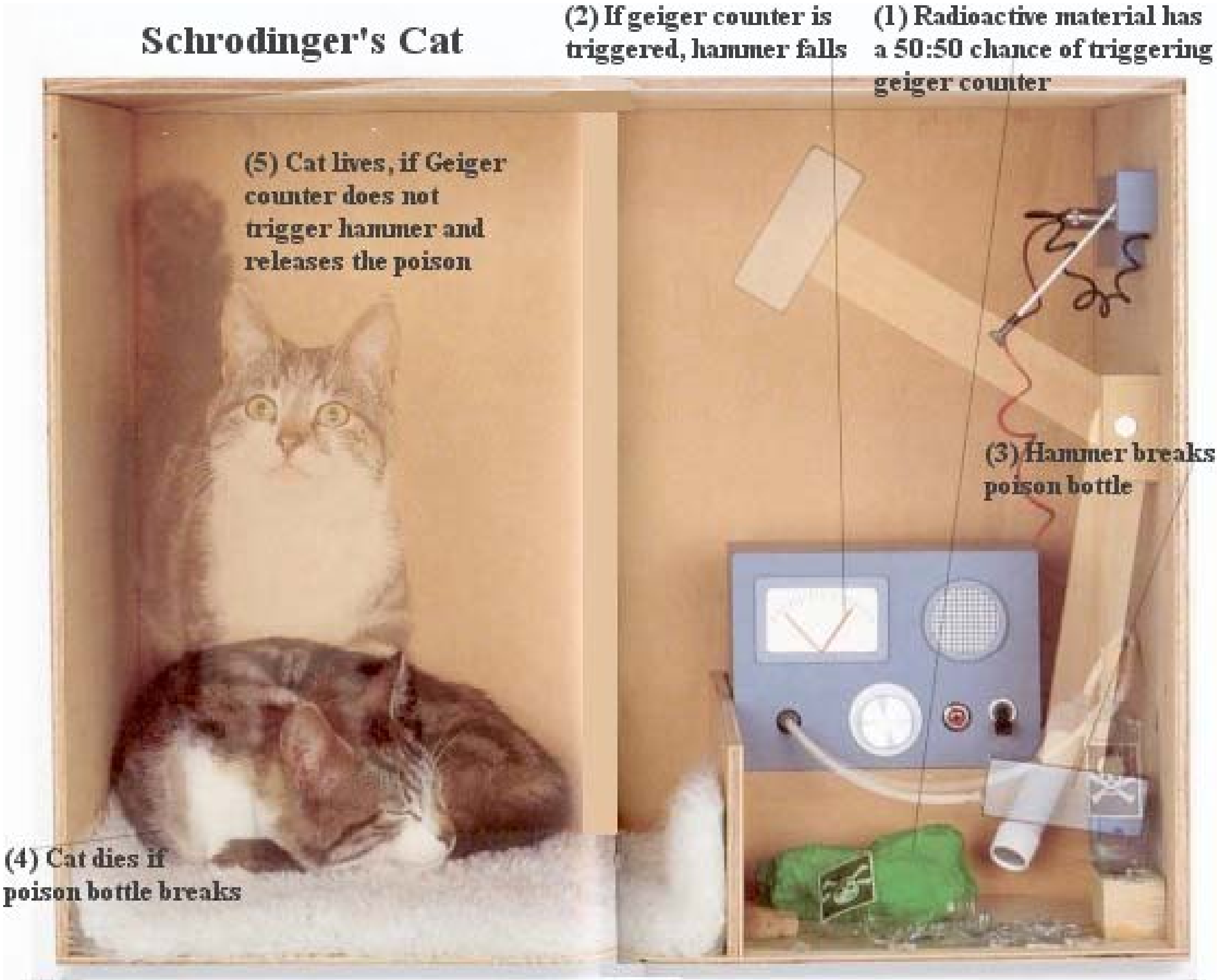
The weirdest feature of the Copenhagen interpretation is that it requires that the wavefunction suffer a discontinuous, unpredictable change during the measurement. Consider, for instance, the impact of an electron on the fluorescent screen in the electron-diffraction experiment. This impact and the flash of light released in it constitute an (approximate) measurement of the position of the electron. Just before this measurement, the wavefunction was spread out all over the screen; immediately after the measurement, the electron position is known to lie within some small spot on the screen, and the wavefunction must therefore have an extent no larger than this spot. Thus, during the measurement, the wavefunction suffers an unpredictable *collapse*, or *reduction*. The collapse is unpredictable, since we have no way of knowing onto what part of the screen the wavefunction will collapse—we know only the probability distribution of the spots on which the wavefunction collapses, that is, the probability distribution of positions for the electron on the screen.

Collapse of the wavefunction and entanglement: Schrodinger's cat

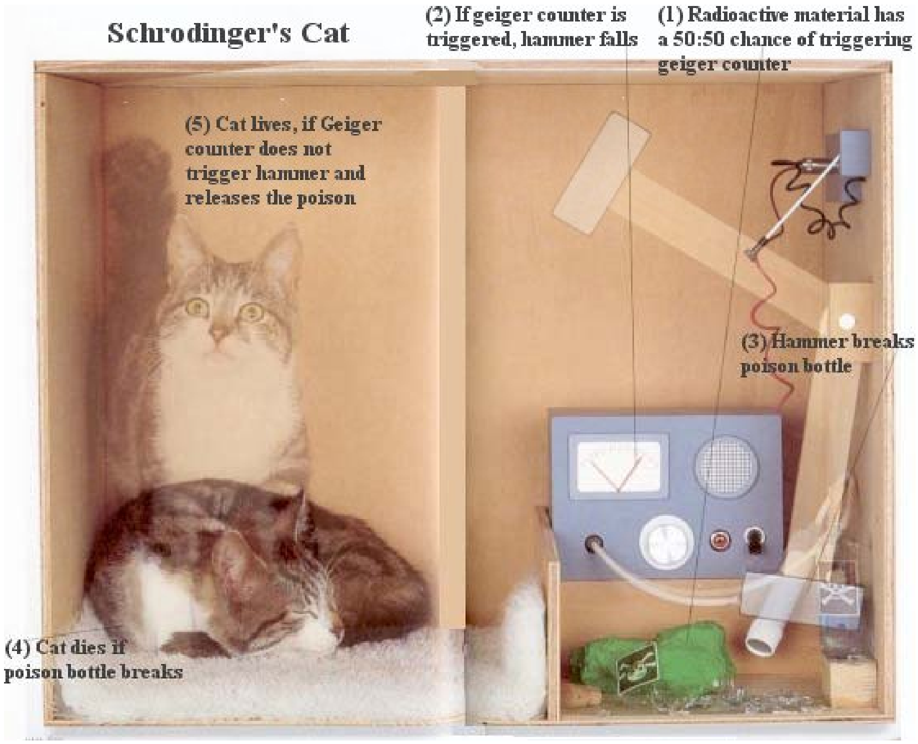
What causes the 'collapse of the wavefunction'? More to the point, how is this effect interpreted?

illustrate with a classical example: Schrodinger's cat

Collapse of the wavefunction and entanglement: Schrodinger's cat



Collapse of the wavefunction and entanglement: Schrodinger's cat



QM: The quantum state of the atom is in a superposed state of a decayed atom state and a not decayed state.

A measurement will collapse this wavefunction into one of these two possibilities, but at what stage of this process does the measurement actually occur?

Interpretations of QM: Griffith's Chapter 1
Go back to detecting a particle which goes through slit

1. The **realist** position: *The particle was at C.* This certainly seems like a sensible response, and it is the one Einstein advocated. Note, however, that if this is true then quantum mechanics is an **incomplete** theory, since the particle *really was* at C, and yet quantum mechanics was unable to tell us so. To the realist, indeterminacy is not a fact of nature, but a reflection of our ignorance. As d'Espagnat put it, "the position of the particle was never indeterminate, but was merely unknown to the experimenter."[3] Evidently Ψ is not the whole story—some additional information (known as a **hidden variable**) is needed to provide a complete description of the particle.

2. The **orthodox** position: *The particle wasn't really anywhere.* It was the act of measurement that forced the particle to "take a stand" (though how and why it decided on the point C we dare not ask). Jordan said it most starkly: "Observations not only *disturb* what is to be measured, they *produce* it. . . . We *compel* [the particle] to assume a definite position."[4] This view (the so-called **Copenhagen interpretation**) is associated with Bohr and his followers. Among physicists it has always been the most widely accepted position. Note, however, that if it is correct there is something very peculiar about the act of measurement—something that over half a century of debate has done precious little to illuminate.

Interpretations of QM: Griffith's Chapter 1

3. The **agnostic** position: *Refuse to answer.* This is not quite as silly as it sounds—after all, what sense can there be in making assertions about the status of a particle *before* a measurement, when the only way of knowing whether you were right is precisely to conduct a measurement, in which case what you get is no longer "before the measurement"? It is metaphysics (in the perjorative sense of the word) to worry about something that cannot, by its nature, be tested. Pauli said, "One should no more rack one's brain about the problem of whether something one cannot know anything about exists all the same, than about the ancient question of how many angels are able to sit on the point of a needle."[5] For decades this was the "fall-back" position of most physicists: They'd try to sell you answer 2, but if you were persistent they'd switch to 3 and terminate the conversation.

Collapse of the wavefunction : Ohanian Chapter 12
Schrodigner's cat scenario:

Although such a schizoid superposition of a live-cat state vector and a dead-cat state vector does violence to our intuition, we cannot disprove it by any experiment. As soon as we open the chamber, or use any measuring device to detect the life signs of the cat, the state vector collapses into either the live-cat configuration or the dead-cat configuration, with equal probabilities. Thus, we can never "see" the cat in the superposed state. Instead, the act of observation or of measurement does something very drastic to the state of the cat—it flips the cat into either the live state or the dead state.

... of a permanent, indelible record.

In the *Gedankenexperiment* of Schrödinger's cat, the orthodox Copenhagen interpretation claims that the quantum-mechanical wavefunction collapses when the Geiger counter makes a measurement on the radioactive substance, and therefore the state of the Geiger counter (and the state of the cat) never forms a superposition of two macroscopically different states. At each instant, the Geiger counter either performs an irreversible act of amplification or does not perform such an act, that is, the Geiger counter adopts either a definite state of discharge or a definite state of no discharge. This means that the Geiger counter acquires information

Interpretations of QM: Ohanian Chapter 12

12.1 *The Copenhagen Interpretation*

The main features of the Copenhagen interpretation can be briefly summarized as follows:

1. The state vector $|\psi\rangle$ provides a complete characterization of the state of the system.
2. The state vector tells us the probability distribution for the result of the measurement of any observable quantity. This probability distribution applies to each *individual* quantum particle or quantum system.
3. The uncertainty relations indicate the intrinsic spreads in the values of complementary observables for the *individual* quantum particle or quantum system. These uncertainty relation deny the existence of sharp values of complementary observables.
4. Measurements produce unpredictable, discontinuous changes in the state vector, which do not obey the Schrödinger equation. The outcome of a single measurement of an observable is unpredictable—the outcome can be any of the eigenvalues within the spread of the probability distribution. During the measurement, the state of the system collapses into an eigenstate of the observable.

Interpretations of QM: Ohanian Chapter 12

Popular Picture. Physicists have a deep predilection for continuity in nature (*Natura non facit saltus*), and they tend to be uncomfortable with the discontinuous collapse and with the somewhat capricious dichotomy between measured system and apparatus demanded by the orthodox Copenhagen picture. The popular picture is an alternative to the orthodox Copenhagen picture; it is favored by many, perhaps by most, of the physicists of today. In the popular picture, there is no collapse. The state vector evolves continuously at all times, according to the Schrödinger equation. Both the system and the apparatus are treated quantum-mechanically, and they are described by a joint state vector. A measurement is regarded as an interaction between the system and the apparatus, as in our example of the Stern–Gerlach experiment of Section 12.2. During such an interaction, the state vectors of the system and the apparatus become correlated, and the joint state vector forms a superposition of these correlated state vectors.

Interpretations of QM: Ohanian Chapter 12

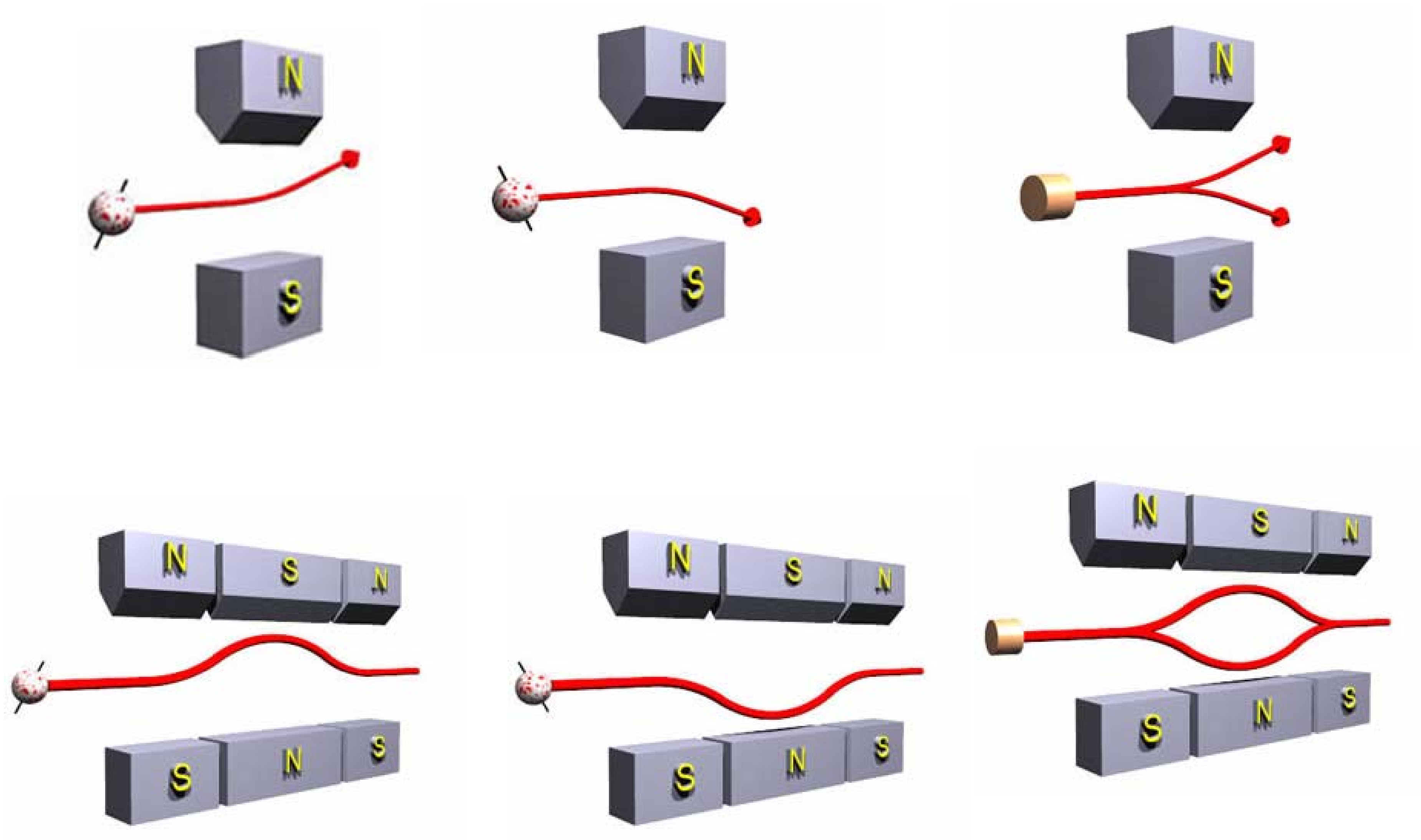
that is, the off-diagonal terms. A somewhat different version of the popular picture attempts to achieve such a cancellation by exploiting unpredictable, random phase differences that are supposedly introduced into the state vector for the system when it interacts with the apparatus during measurement.[12] This version of the popular picture argues that the microscopic quantum state of the apparatus is not known, and is not reproducible from one repetition of the measurement to the next; even if we "reset" the apparatus for each repetition of the measurement, there will be uncontrollable and unpredictable fluctuations in its microscopic quantum state. When the measured system interacts with this ap-

Interpretations of QM: Ohanian Chapter 12

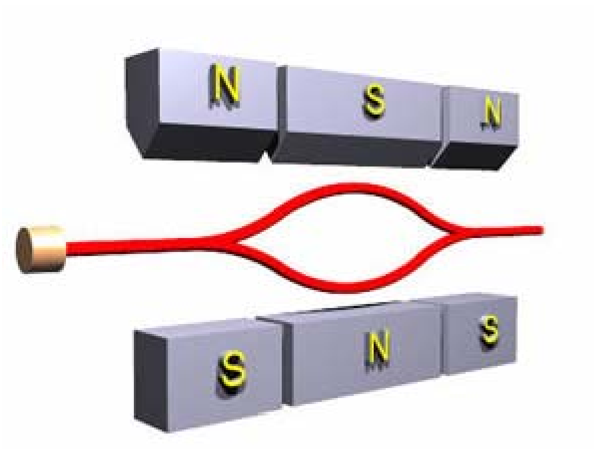
Subjective Picture. Another proposal for the collapse is that it is produced in the mind of the observer, by the intervention of the observer's consciousness. This notion was first proposed by von

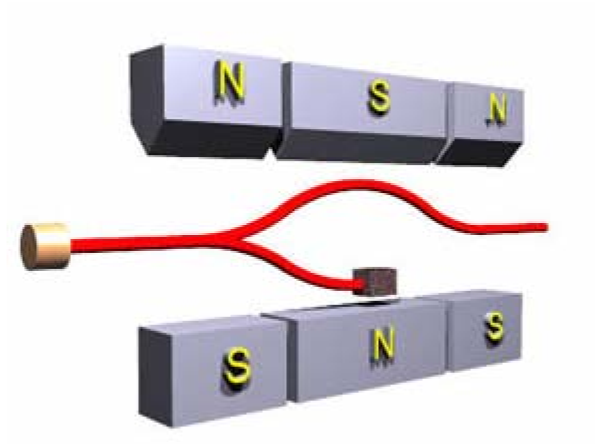
Many-Worlds Picture. Another, radically different treatment of the collapse problem is the many-worlds picture of Everett.[15] In this picture, as in the popular picture, there is no collapse, and the state vector evolves according to the Schrödinger equation at all times. But the many-worlds picture differs from the popular picture in that it includes the observer as part of the quantum-mechanical system. Thus, the many-worlds picture eliminates the dividing line (Heisenberg cut) between the observer and the apparatus, whereas the popular picture implicitly retains this dividing line. The interaction between measured system, apparatus, and

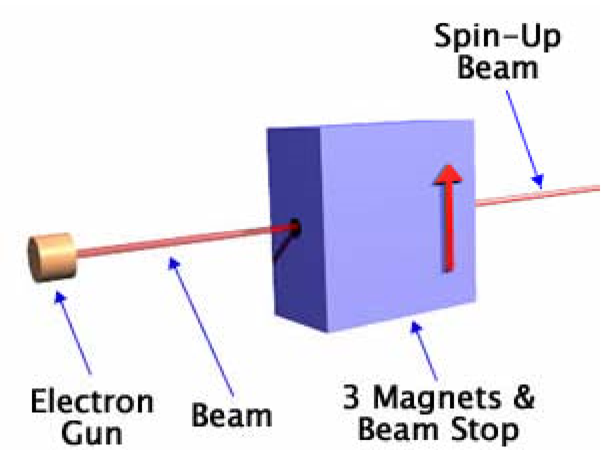
Stern-Gerlach Experiment with electrons:



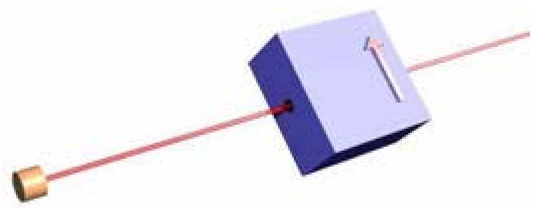
Stern-Gerlach Filter:

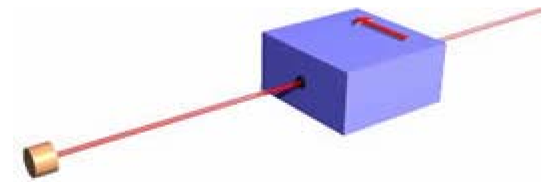


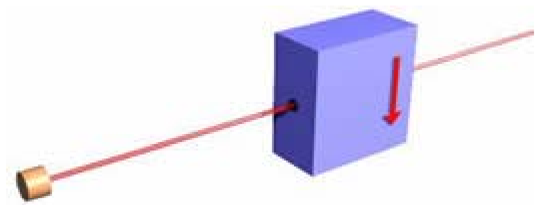


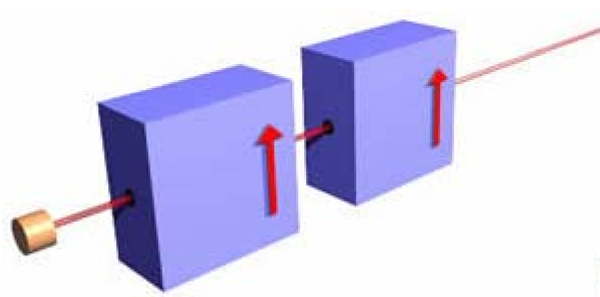


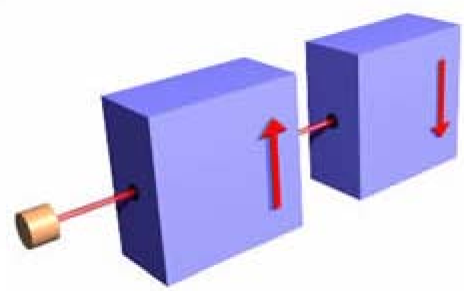
Using the Stern-Gerlach Filters:

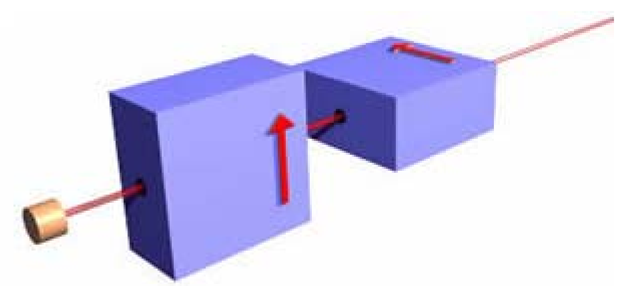






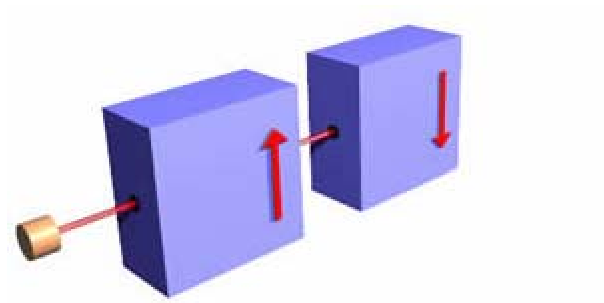


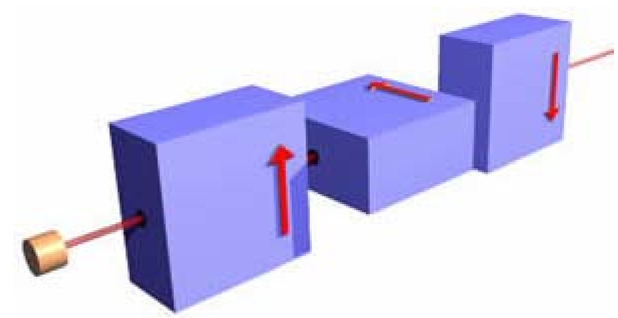




Using the Stern-Gerlach Filters:

http://faraday.physics.utoronto.ca/PVB/Harrison/SternGerlach/Flash/SGInteractive.html



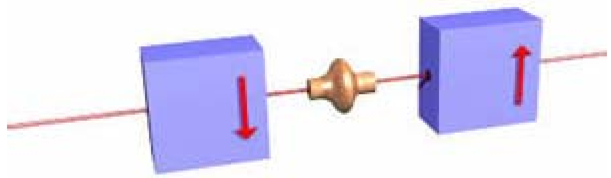


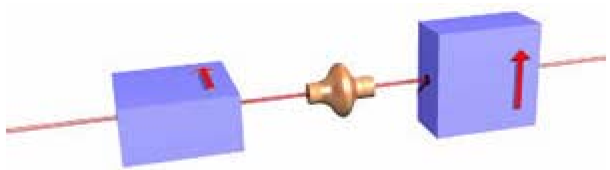
Fraction transmitted from one SG through the next:

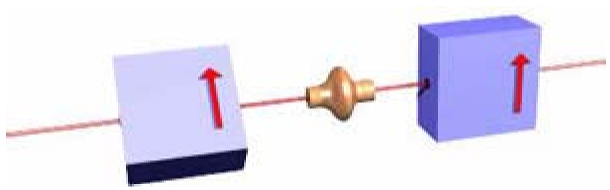
$\cos(a/2)^2$

where *a* is the relative angle between the filters

Correlated Electrons + Stern Gerlach (SG)







Consider a radioactive substance that emits a pair of electrons in each decay.

Conservation of momentum:

1. two electrons travel away from each other
2. They will have opposite angular momentum (spins) i.e. the spins are correlated

the correlation is sin(*a*/2)^2 squared where *a* is the relative angle between the filters

Special Notes:

One measurement does not effect the statistics of the other measurement

The statistics of both measurements are effected by the relative orientation!

Bell’s Inequality using Stern-Gerlach filters:

Using the Stern-Gerlach Filters:

... of electric charge in space.

Quantum mechanics does not supply us with concrete mental pictures of the behavior of atoms and subatomic particles. Quantum mechanics does not tell us what atoms and subatomic particles are like; it merely tells us what happens when we perform measurements. As Heisenberg said: "The conception of objective reality . . . evaporated into the . . . mathematics that represents no longer the behavior of elementary particles but rather our knowledge of this behavior."[1]

Interpretations of QM:

...not to the agencies of observation."[4]

According to the Copenhagen interpretation, quantum systems in themselves do not have sharply defined attributes, only diffuse potentialities, which are capable of becoming sharply defined when we perform suitable measurements. The attributes of a quantum system depend on the apparatus used to measure them, and they exist only in relation to this apparatus. Thus, the attributes are a joint property of the system and the apparatus. This