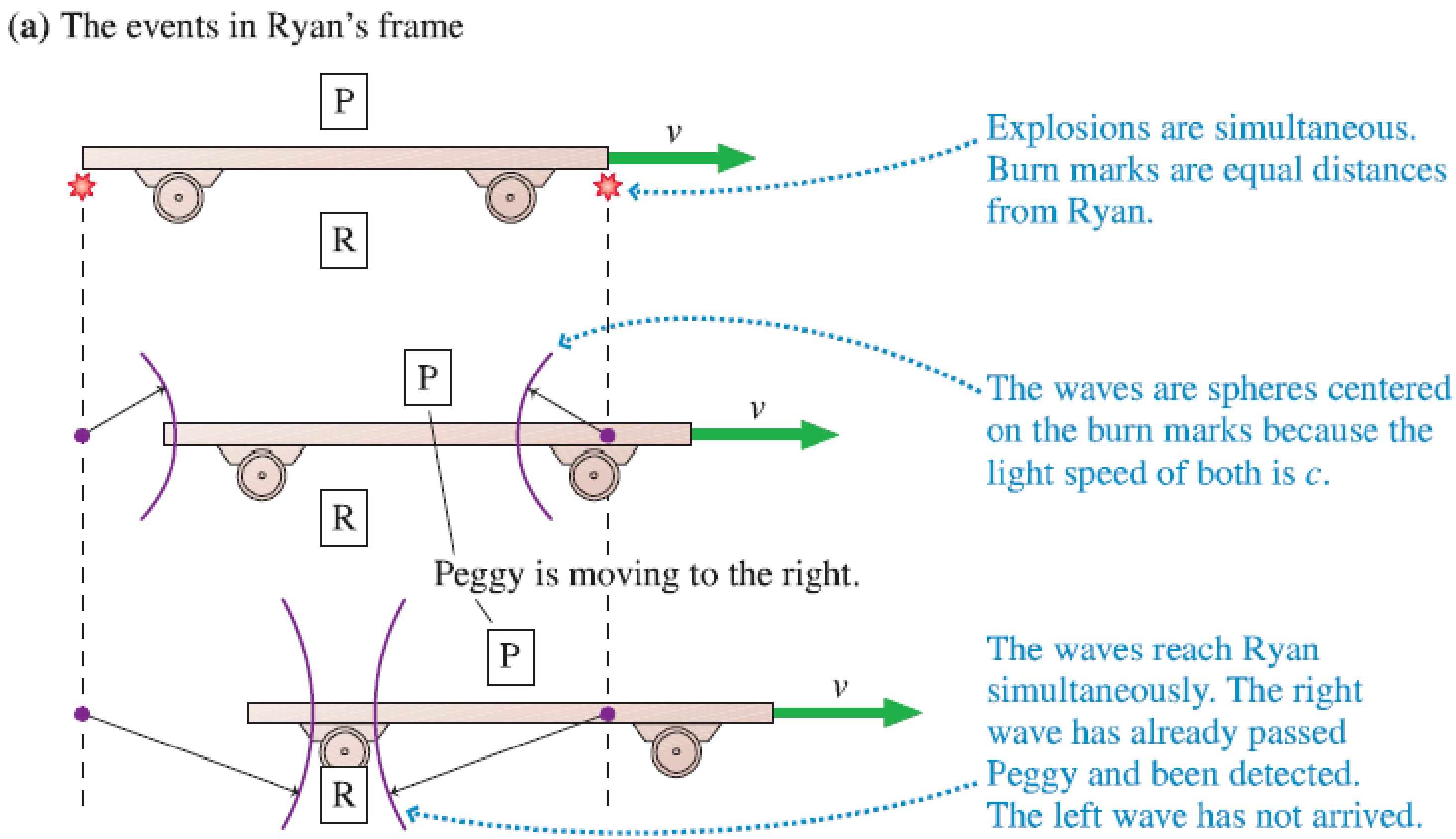
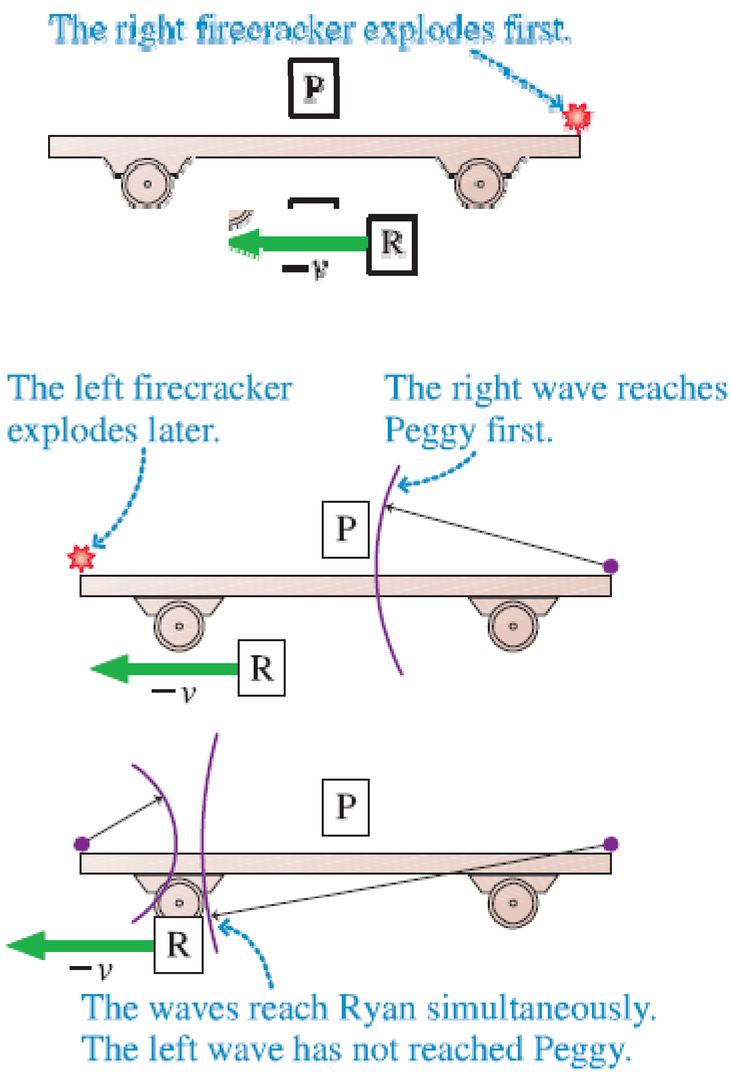
Exam III review

Ryan's Reference frame analysis



Light is Green since "right" firecracker light reaches detector first.

Peggy's Reference frame Correct analysis



Ryan ***must*** detect the two waves simultaneously. Everything flows from this idea.

Since the wave from the right firecracker must travel further to reach Ryan IN PEGGY's FRAME, it must have exploded before the left firecracker IN PEGGY's FRAME.

The firecrackers are NOT simultaneous in Peggy's frame, although they are in Ryan's frame

The light is green.

"simultaneity" is relative --- that is, whether two events occur at the same time is dependent upon your reference frame

(b) The clock is at rest in frame S′.

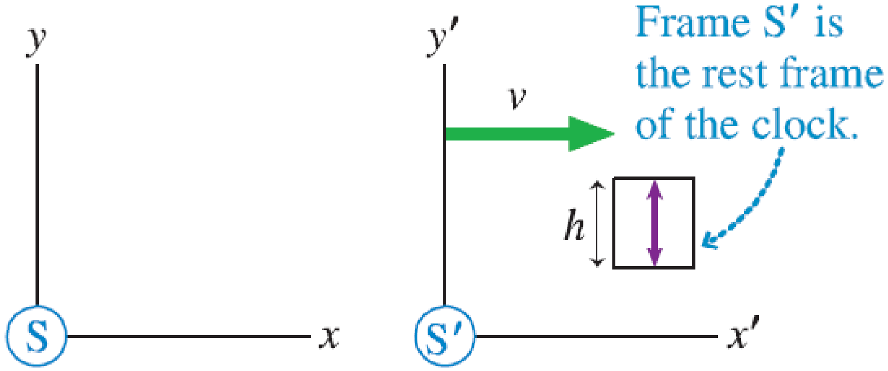
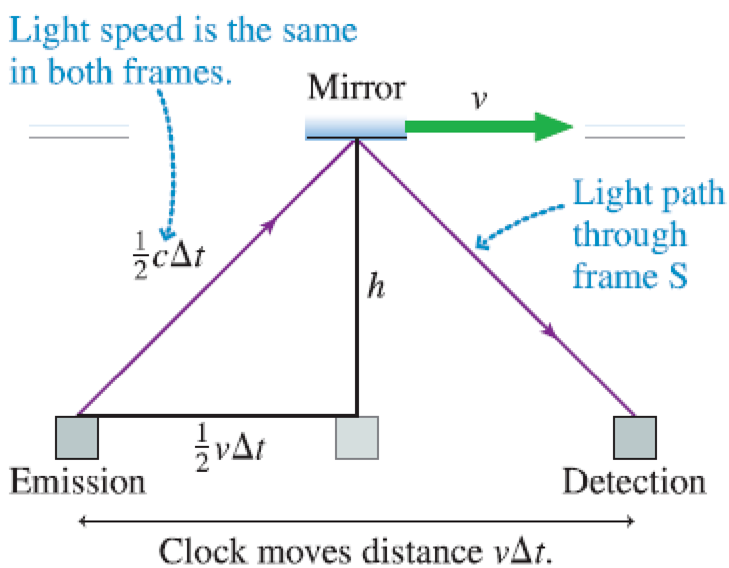


FIGURE 37.21 A light clock analysis in which the speed of light is the same in all reference frames.



frame S′

$$c = \frac{\Delta l'}{\Delta t'} \Rightarrow \Delta t' = \frac{2h}{c}$$

frame S:

$$c = \frac{\Delta l}{\Delta t} \; ; \; \Delta l = 2\sqrt{h^2 + \left(\tfrac{1}{2} v \Delta t\right)^2}$$

$$\Rightarrow \Delta t = \frac{\Delta l}{c} \Rightarrow \Delta t^2 = \frac{1}{c^2}\left[(2h)^2 + (v\Delta t)^2\right]$$

$$\Rightarrow (\Delta t)^2\left(1 - \Big(\underbrace{\tfrac{v}{c}}_{\beta}\Big)^2\right) = \left(2h/c\right)^2$$

$$\Rightarrow \Delta t = \left(1-\beta^2\right)^{-1/2}\left(2h/c\right)$$

$$\therefore \Delta t = \frac{\Delta t'}{\sqrt{1-\beta^2}} \; , \; \beta \equiv \frac{v}{c}$$

(b) The clock is at rest in frame S′.

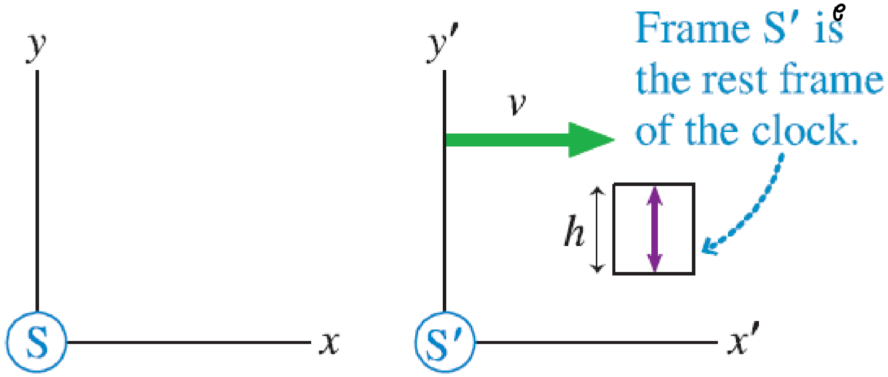
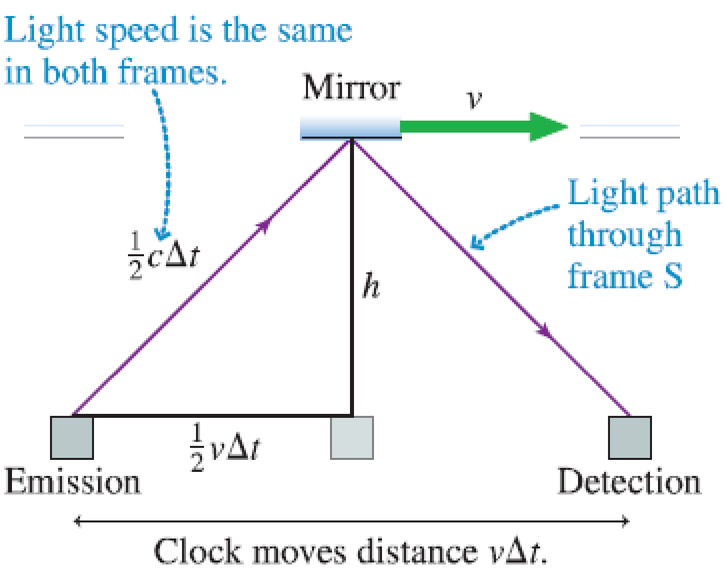


FIGURE 37.21 A light clock analysis in which the speed of light is the same in all reference frames.



$$\therefore \Delta t = \frac{\Delta t'}{\sqrt{1-\beta^2}}, \quad \beta \equiv \frac{v}{c}$$

Shortest time between ticks is in the frame where the clock is at rest --- That is, the frame in which the two events (emission and detection) are measured with the ***same*** clock. In this case, this is called the proper time and is notated as $\Delta\tau$.

MORE time passes per tick in frame S in which the clock is moving than in the stationary frame S'.

$$\Delta t = \frac{\Delta \tau}{\sqrt{1-\beta^2}} \geq \Delta \tau \qquad \text{(time dilation)}$$

"time dilation"

Twin Paradox

Two twins, call them Earl and Roger:
Earl is on Earth
Roger is in Rocket

Roger takes off at relativistic velocity to Jupiter and back. Both Roger and Earl measure the take off event and the return event with the ***same*** clock in their respective reference frames.

Who is it that is measuring the proper time? Both Roger and Earl think they are measuring Proper time and think that the other guy should be younger (slower clock) than themselves.

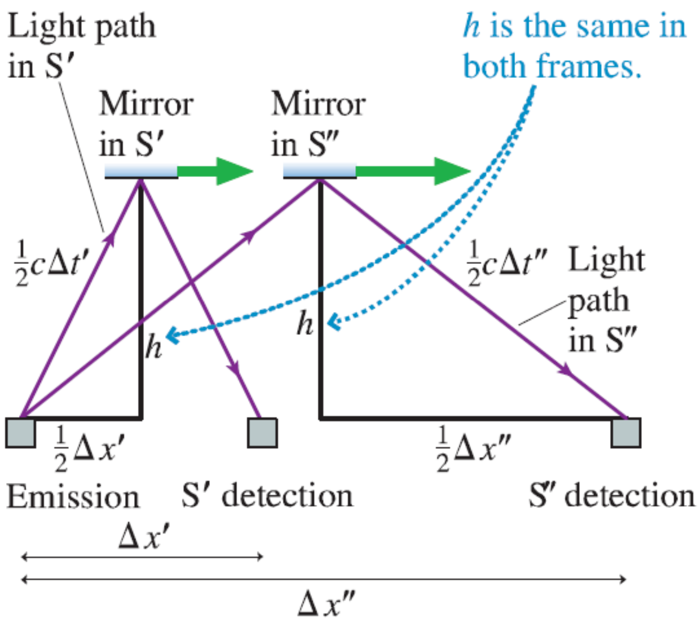
There is another intermediary event: the rocket decelerates and accelerates to turn around and go back to earth. Since this event is not measured by Earl with the same clock, Earl is not measuring proper time. Roger measures proper time. Therefore Roger is younger than Earl upon his return.

Caveat: The 'lost' time must be associated with the acceleration and deceleration….

Length contraction derived from Railroad car (see pdf Notes)

Space-time interval

FIGURE 37.28 The light clock seen by experimenters in reference frames S′ and S″.



h is invariant no matter how fast the reference frame is moving

$$h^2 = \left(\frac{1}{2}c\,\Delta t'\right)^2 - \left(\frac{1}{2}\Delta x'\right)^2 = \left(\frac{1}{2}c\,\Delta t''\right)^2 - \left(\frac{1}{2}\Delta x''\right)^2$$

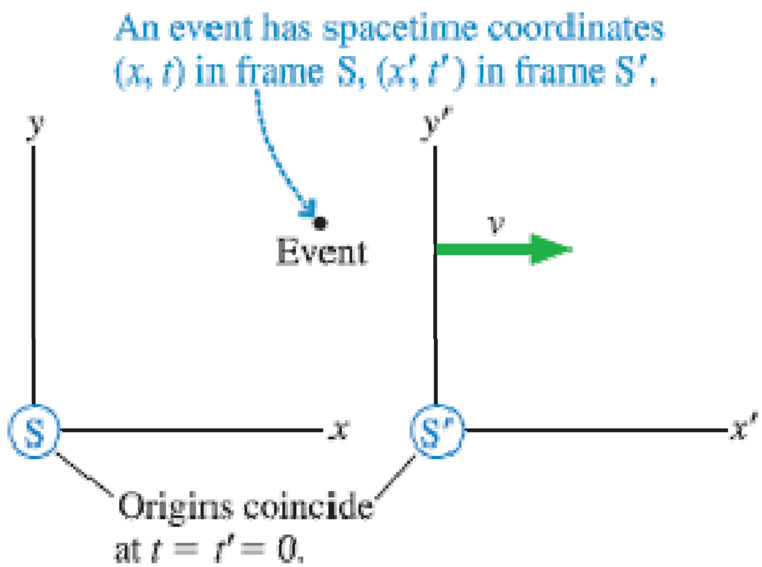
$$c^2(\Delta t')^2 - (\Delta x')^2 = c^2(\Delta t'')^2 - (\Delta x'')^2$$

spacetime interval s

$$s^2 = c^2(\Delta t)^2 - (\Delta x)^2$$

S is an invariant in relativity --- all observers will measure the same spacetime interval between two events

Lorentz transformation



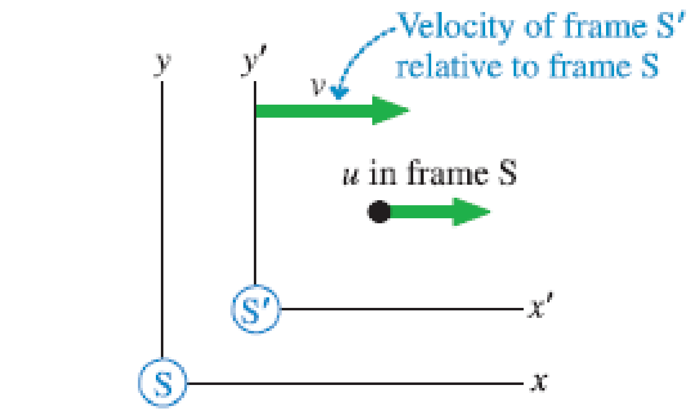
$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$

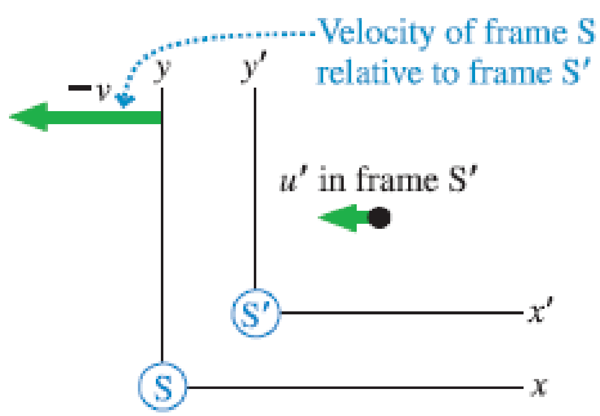
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

The Lorentz transformations transform the spacetime coordinates of *one* event. Compare these to the Galilean transformation equations in Equations 37.1.

Lorentz velocity transformation

FIGURE 37.33 The velocity of a moving object is measured to be u in frame S and u' in frame S'.





$$u = \frac{dx}{dt} \quad + \quad u' = \frac{dx'}{dt'}$$

Relationship between u + u' ?

$$x' = \gamma(x - vt) \;,\; t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$\Rightarrow dx' = \gamma(dx - v\,dt) , dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$$

$$\therefore u' = \frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - \frac{v}{c^2}dx} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

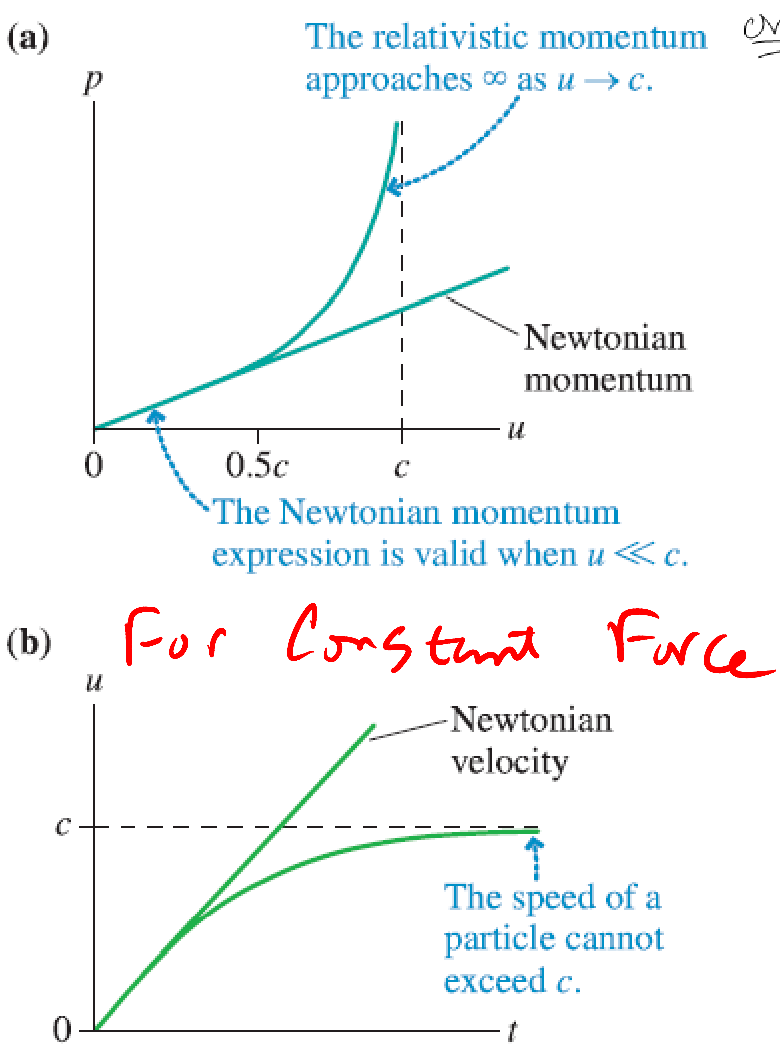
$$\Rightarrow u' = \frac{u - v}{1 - \frac{v}{c^2}u}$$

Similarly: $x = \gamma(x' + vt')$, $t = \gamma\left(t' + \frac{v}{c^2}x'\right)$

$$\Rightarrow u = \frac{u' + v}{1 + \frac{v}{c^2}u'}$$

Relativistic Momentum

FIGURE 37.34 The speed of a particle cannot reach the speed of light.



$p = \gamma_p m u$ where $\gamma_p = \dfrac{1}{\sqrt{1 - u^2/c^2}}$

or

$p = m_{eff}\, u$ where $m_{eff} = \dfrac{m_0}{\sqrt{1 - u^2/c^2}}$

& m_0 = Rest mass

For Constant Force,

$p = F \cdot t$

$m_{eff}\, u = F t$

Relativistic Energy

Let a particle of mass m move through distance Δx during a time interval Δt, as measured in reference frame S. The spacetime interval is

$$s^2 = c^2(\Delta t)^2 - (\Delta x)^2 = \text{invariant}$$

We can turn this into an expression involving momentum if we multiply by $(m/\Delta\tau)^2$, where $\Delta\tau$ is the proper time (i.e., the time measured by the particle). Doing so gives

$$(mc)^2\left(\frac{\Delta t}{\Delta\tau}\right)^2 - \left(\frac{m\Delta x}{\Delta\tau}\right)^2 = (mc)^2\left(\frac{\Delta t}{\Delta\tau}\right)^2 - p^2 = \text{invariant} \qquad (37.37)$$

where we used $p = m(\Delta x/\Delta\tau)$ from Equation 37.32.

Now Δt, the time interval in frame S, is related to the proper time by the time-dilation result $\Delta t = \gamma_p \Delta\tau$. With this change, Equation 37.37 becomes

$$(\gamma_p mc)^2 - p^2 = \text{invariant}$$

Finally, for reasons that will be clear in a minute, we multiply by c^2, to get

$$(\gamma_p mc^2)^2 - (pc)^2 = \text{invariant} \qquad (37.38)$$

Relativistic Energy

$$(\gamma_p mc^2)^2 - (pc)^2 = \text{invariant}$$

$$\underbrace{(\gamma_p mc^2)^2 - (pc)^2}_{\text{frame } S} = \underbrace{(\gamma_p' mc^2)^2 - (p'c)^2}_{\text{frame } S'}$$

$$(\gamma_p mc^2)^2 - (pc)^2 = \underbrace{(mc^2)^2}_{\text{"Particle at Rest" frame}}$$

$(p' = 0 \Rightarrow \gamma_p' = 1)$

Relativistic Energy

$$(\gamma_p mc^2)^2 - (pc)^2 = \text{invariant}$$

$$\gamma_p mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} \approx \left(1 + \frac{1}{2}\frac{u^2}{c^2}\right)mc^2 = mc^2 + \underbrace{\frac{1}{2}mu^2}_{KE}$$

$u \ll c$

New!

An inherent energy associated with the particles rest mass!

Call it "rest" Energy of particle

$$(\gamma_p mc^2)^2 - (pc)^2 = (mc^2)^2$$

Relativistic Energy

$$\gamma_p mc^2 = \frac{mc^2}{\sqrt{1 - u^2/c^2}} \approx \left(1 + \frac{1}{2}\frac{u^2}{c^2}\right)mc^2 = mc^2 + \frac{1}{2}mu^2$$

Define E, KE, & rest Energy E_0

$$E = \gamma_p mc^2 = E_0 + K = \text{rest energy} + \text{kinetic energy}$$

This total energy consists of a **rest energy**

$$E_0 = mc^2$$

and a relativistic expression for the *kinetic energy*

$$K = (\gamma_p - 1)mc^2 = (\gamma_p - 1)E_0$$

$$E^2 - (pc)^2 = E_0^2$$

State of 19th and very early 20th century physics:

Light:

1. E&M **Maxwell's equations** --> waves; J. J. Thompson's **double slit experiment** with light

2. Does light need a medium? --> Aether and **Michelson-Moreley experiment**
   1 and 2 lead to **Relativity**, 1905

3. Detected in discrete lumps --> **photoelectric effect**
   concept of photons

4. **Black Body radiation spectrum** -- Planck's proposition that **energy is quantized**

5. Gas discharge tube produces **discrete line spectrum** which depends upon atoms vs. black body radiation (incandescence) continuous spectrum

6. Emission spectra of hydrogen fairly simple (**Balmer formula**)

State of 19th and early 20th century physics:

Matter:

1. J. J. Thompson measures q/m of 'cathode rays'---> **crossed-field experiment**

2. J. J. Thompson used Rontgen's x-rays to ionize helium, and the same q/m produced; Also e/m from hot wire same q/m ---> q=e, atoms composed of subatomic particles called 'electrons'

3. J. J. Thompson measures the q/m ratio of hydrogen ion which is MUCH smaller than electron

4. **Millikan oil drop experiment**: measures discrete charge e ---> e of electron (& m of electron)

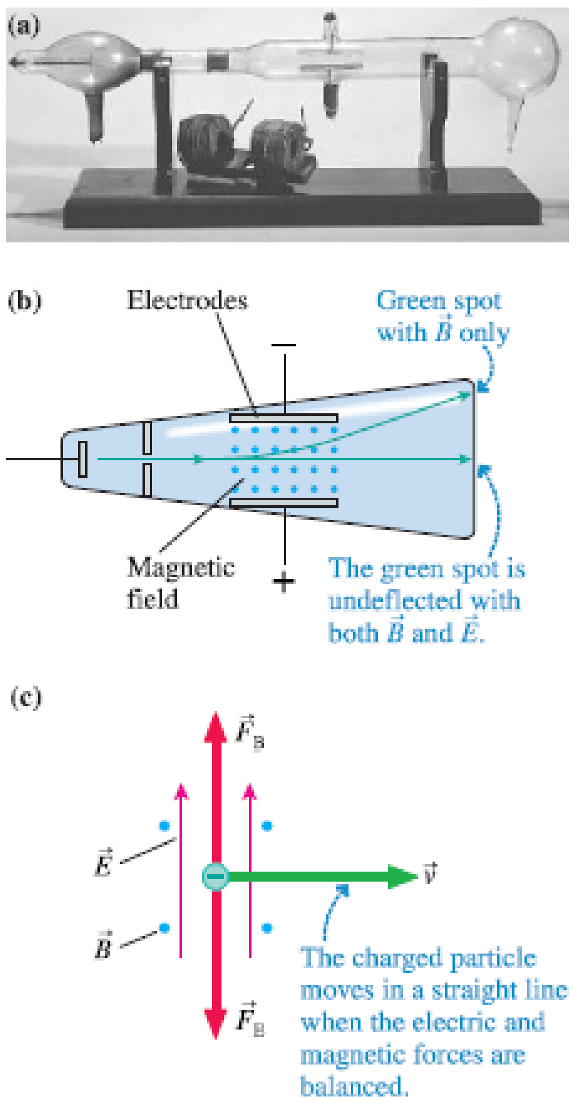
5. Rutherford , uranium decay produces "beta rays" (high speed electrons) and "alpha rays"; alpha rays trapped in gas discharge tube and produced same discrete spectrum as Helium + measurement of q/m of alpha --> double ionized He ions; Uranium emitting other particles, He and electrons

6. **Rutherford's foil experiment**, fires doubly ionized He at gold foil, most go through but some bounce back --> requires positively charged, heavy centers in gold foil --> nucleus surrounded by 'orbiting' electrons; Deduces the rough diameter of nucleus ~ 10 fm (10^-15 m)

7. 1910, J. J. Thompson develops **mass spectrometer** --> same element has different masses; first evidence of 'something else' besides electrons and positively charged atoms, leading to the discovery of neutrons

J. J. Thompson's measurement of e/m

FIGURE 38.7 Thomson's crossed-field experiment to measure the velocity of cathode rays. The photograph shows his original tube and the coils he used to produce the magnetic field.



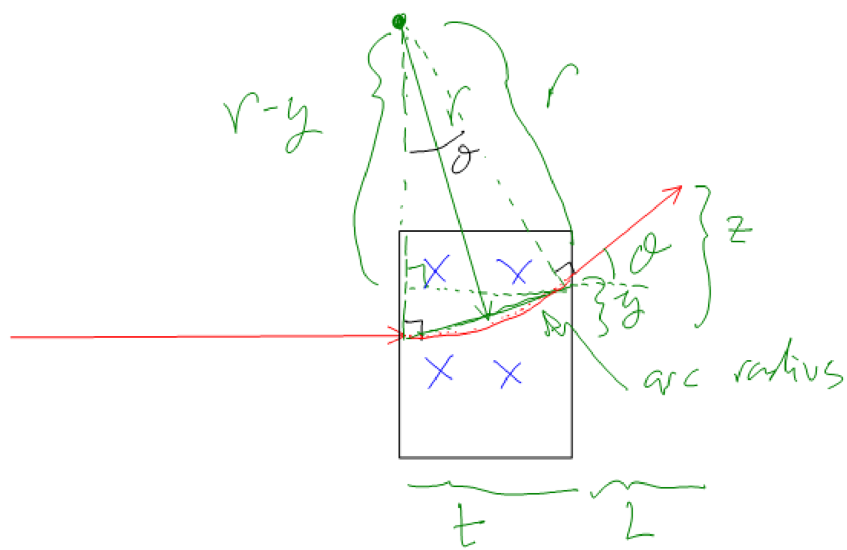
Case I: F_E cancels F_B gives v

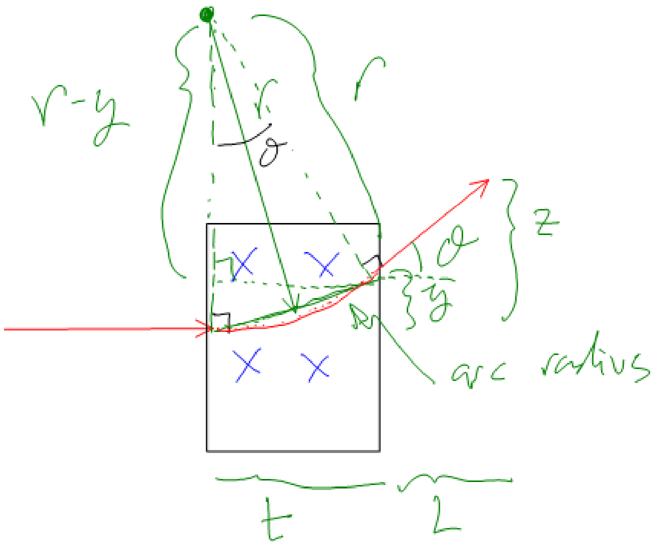
$$F_E = F_B \Rightarrow q\vec{E} = q\vec{v}\times\vec{B}$$

$$\Rightarrow qE = qvB$$

$$\Rightarrow v = \frac{E}{B}$$

Case II: E field off, B field on, measure radius of curvature of arc in B-field





Case II: E field off, B field on, measure radius of Curvature of arc in B-field

$$F = m\frac{v^2}{r} = qvB \quad , \quad v = \frac{E}{B} \text{ from Case I}$$

$$\Rightarrow \frac{q}{m} = \frac{v}{rB}$$

$$(r-y)^2 + t^2 = r^2 \quad , \quad \tan\vartheta = \frac{t}{(r-y)} = \frac{z-y}{L} \Rightarrow (r-y) = \frac{tL}{(z-y)}$$

Measure, t, L, z to find r

$$\therefore r^2 = t^2 + \left(\frac{tL}{z-y}\right)^2$$

for small deflection
$t \ll L$, $z \ll L$
$y \ll z$

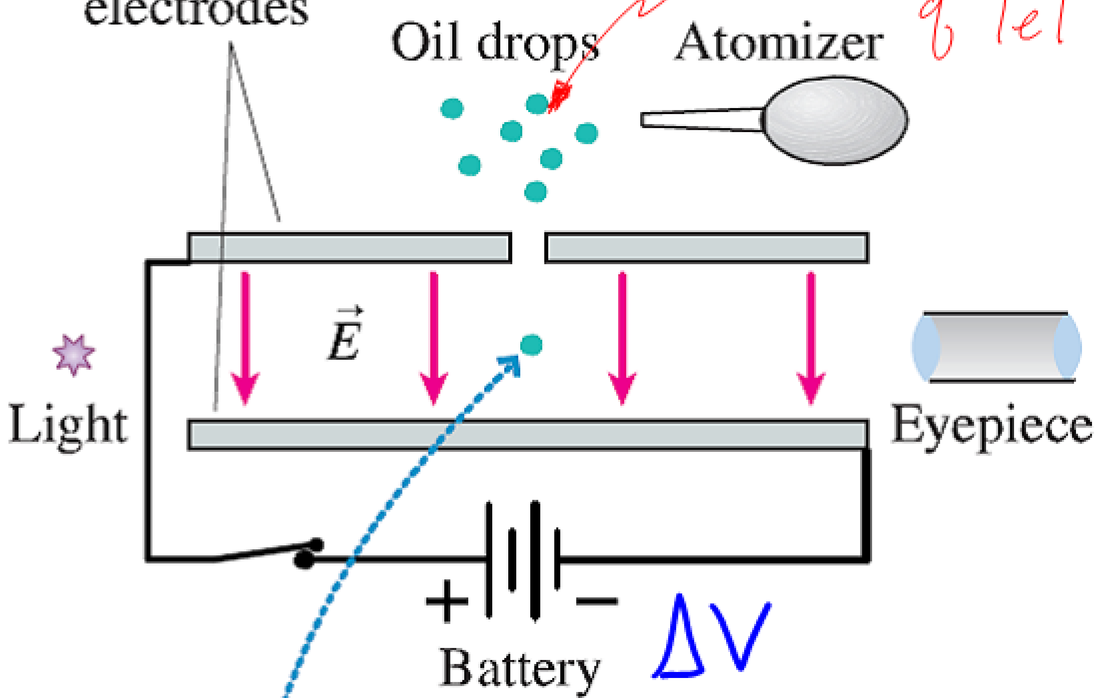
$$\Rightarrow r^2 = t^2\left[1 + \left(\frac{L}{z}\right)^2\right]$$

$$\Rightarrow r^2 = \frac{t^2}{z^2}\left[z^2 + L^2\right]$$

$$\Rightarrow r \approx \frac{t}{z} L$$

Millikan's Oil drop experiment: Measuring quantized q

oil drops charged to integer values $q\,|e|$



The upward electric force on a negatively charged droplet balances the downward gravitational force.

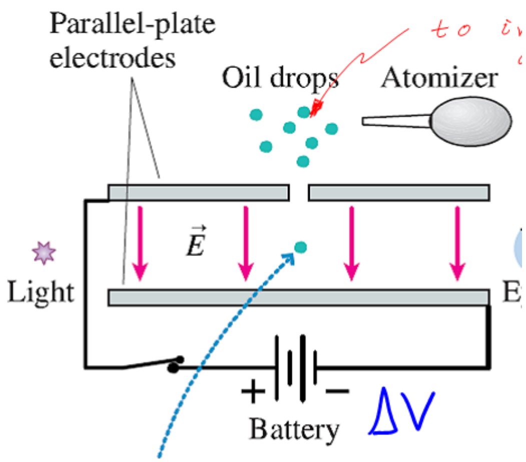
Three forces on oil

① Drag force when moving @ terminal velocity $= C\vec{v}^2$; C a constant

② Gravity, $m\vec{g}$

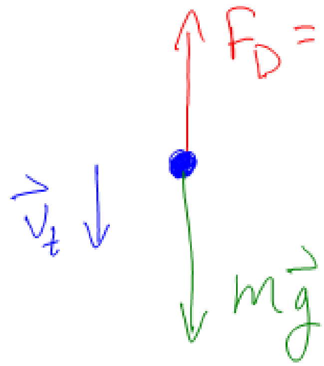
③ Electric force, $q\vec{E}$

Millikan's Oil drop experiment: Measuring quantized q



I. Battery off:

$$\vec{F}_D = C\vec{v}_t^{\,2}$$



at terminal velocity:

$$F_D = mg \Rightarrow C = \frac{mg}{V_t^2}$$

V_t is measured

I. Battery off:

at terminal velocity:

$$F_D = mg \Rightarrow c = \frac{mg}{V_t^2}$$

II. Battery on:

at terminal velocity:

$$F_D' + mg = qE \,, \quad E = \Delta V/d \,, \; d = \text{plate separation}$$

$$\underbrace{c v'^2}_{= mg \frac{v'^2}{v^2}} + mg = q\, \Delta V/d$$

$$\Rightarrow mg\left(\frac{v_t'^2}{v_t^2} + 1\right) = q\,\frac{\Delta V}{d}$$

$$\Rightarrow q = \frac{mgd}{\Delta V}\left(\frac{v_t'^2}{v_t^2} + 1\right)$$

radius of oil droplet

$$m = \text{mass of oil droplet} = \rho\left(\frac{4}{3}\pi a^3\right)$$

density of oil

$$\Rightarrow q = \left(\frac{4}{3}\pi g\right)\left(\rho\, d\right) \underbrace{\frac{a^3}{\Delta V}\left(\frac{v_t'^2}{v_t^2} + 1\right)}$$

Constants

Measured before expt.,

ΔV, v_t' + V_t measured during experiment

"a" is difficult to measure

$$q = \left(\frac{4}{3}\pi g\right)(\rho\, d)\, \underbrace{\frac{a^3}{\Delta V}\left(\frac{V_t'^2}{V_t^2} + 1\right)}_{\Delta V,\ V_t'\ \&\ V_t \text{ measured during experiment}}$$

Constants

Measured before expt.,

"a" is difficult to measure

$$F_D = \frac{C_d}{2}\, \rho_A\, V_t^2\, A_\perp \,, \quad A = \text{cross sectional area} = \pi\left(\frac{a}{\cancel{2}}\right)^2, \quad a = \text{~~diameter~~ radius of oil drop}$$

C_d = drag coefficient in air

ρ_A = density of air

For Case I:

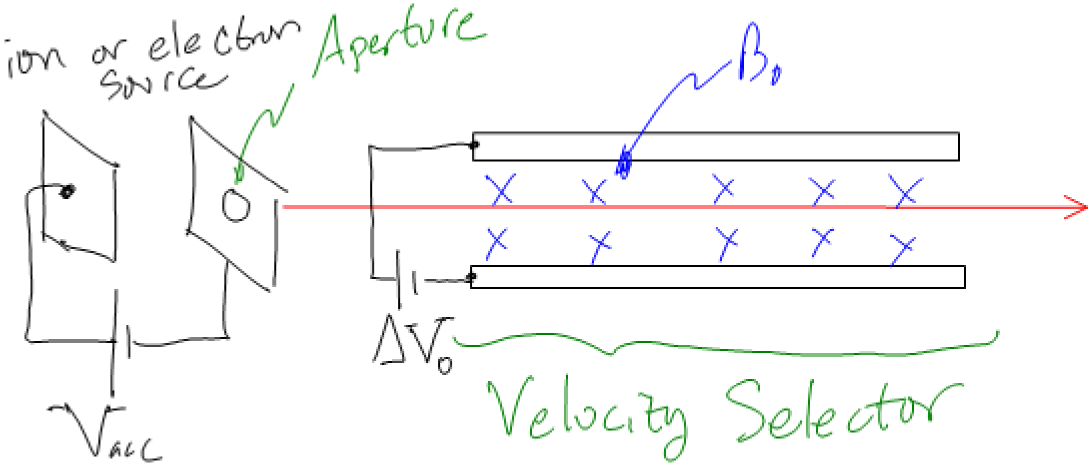
$$F_D = mg \Rightarrow \frac{C_d}{2\cancel{8}}\, \rho_A V_t^2\, \pi a^2 = \rho_{oil}\, \frac{4}{3}\pi\left(\frac{a}{\cancel{2}}\right)^3 g$$

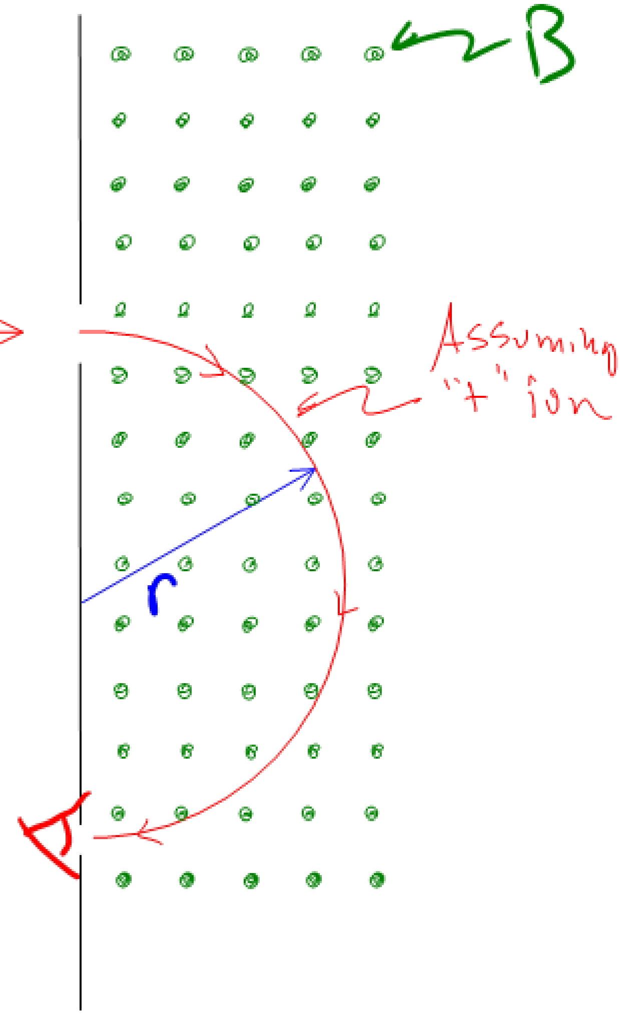
$$\Rightarrow a = \frac{3}{8}\, \frac{C_d}{g}\, \frac{\rho_A}{\rho_{oil}}\, V_t^2$$

$$\therefore\ q = \left(\frac{4}{3}\pi g\right)(\rho\, d)\, \frac{a^3}{\Delta V}\left(\frac{V_t'^2}{V_t^2} + 1\right)$$

Measured: $q = |ne|$ for all oil drops, n integer

J. J. Thompson's mass spectrometer: First evidence of neutron





ion source:

$$\frac{1}{2}mv^2 \approx qV_{acc} \Rightarrow \frac{q}{m} = \frac{1}{2}\frac{v^2}{V_{acc}}$$

spread in v's

Velocity Selector:

$$F = qE_0 = qvB_0, \quad E = \frac{\Delta V_0}{d}$$

$$\Rightarrow v = \frac{E_0}{B_0} = \frac{\Delta V_0}{B_0 d}$$

J. J. Thompson's mass spectrometer: First evidence of neutron

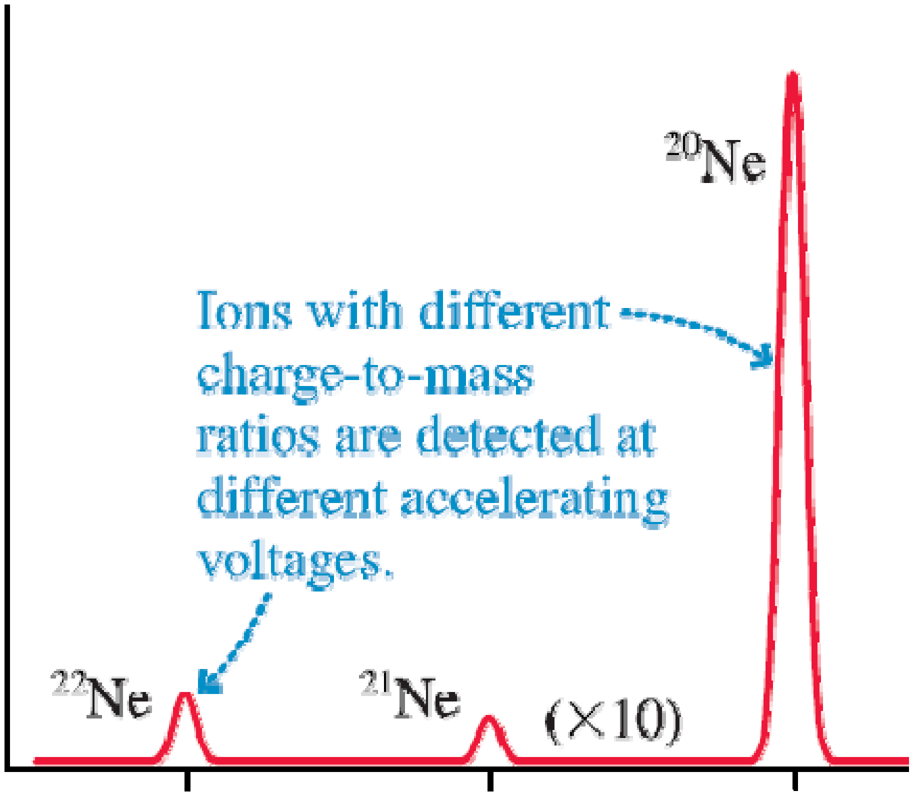
$$v = \frac{E_0}{B_0} = \frac{\Delta V_0}{B_0 d}$$

$$F = m\frac{v^2}{r} = qvB \implies m = \frac{rqB}{v} = q\left[\frac{rBB_0 d}{\Delta V_0}\right]$$

Know $q = |ne|$

So $m = n\left(\frac{|e| rBB_0 d}{\Delta V_0}\right)$

Discovery of isotopes



Some definitions

Definition of eV: energy required to move 1 electron across 1 V = 1.6 x 10^-19 J

Recall: $W = q\Delta V$ so $\text{Energy} = \overset{1.6 \cdot 10^{-19} C}{e}(1V) = 1.6 \cdot 10^{-19} J$

atomic number: number of electrons or protons, Z

Mass number: A=Z+N, N is the number of neutrons

Einstein's photon postulate

Relativity: $E^2 - (pc)^2 = (mc^2)^2$

Anything traveling @ speed of light $c \Rightarrow m = 0$

$$\Rightarrow \quad E = pc$$

Postulate: $E_{ph} = \hbar\omega \Rightarrow E_{ph} = hf$

$$\Rightarrow \quad p_{ph} = \frac{E_{ph}}{c} = h\frac{f}{c}, \quad \& \quad c = \lambda f$$

$$\Rightarrow \quad p_{ph} = \frac{h}{\lambda}$$

Note: Postulate includes that energy of E&M wave is quantized!

Proof of photons: Photoelectric effect

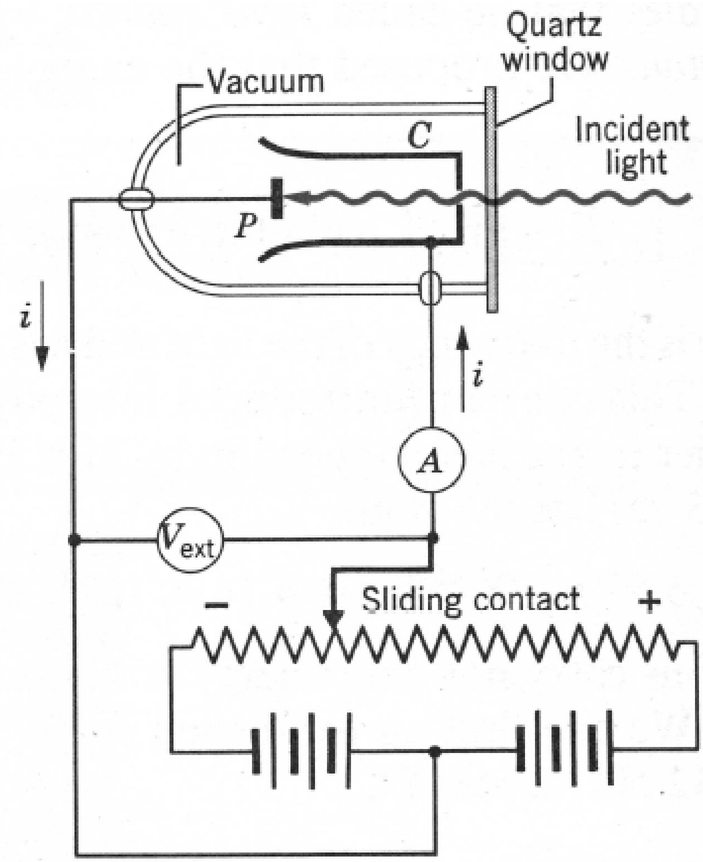
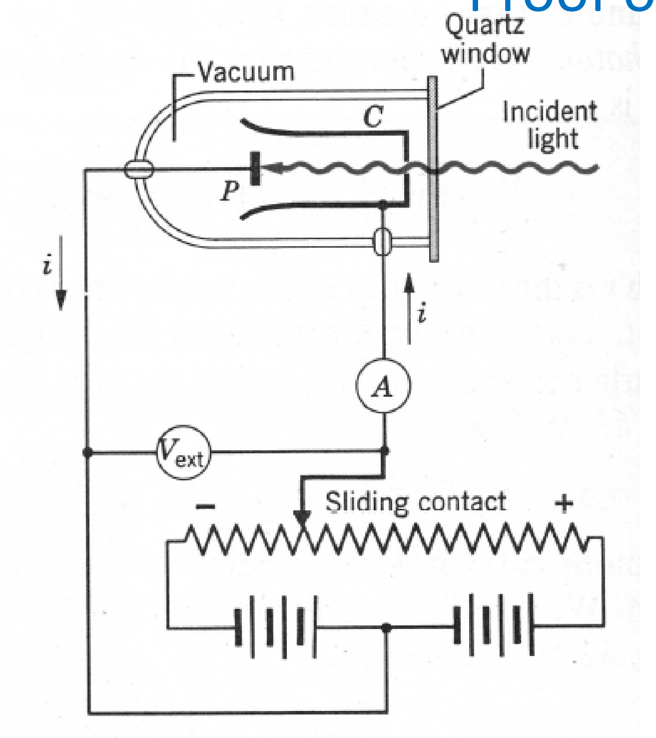
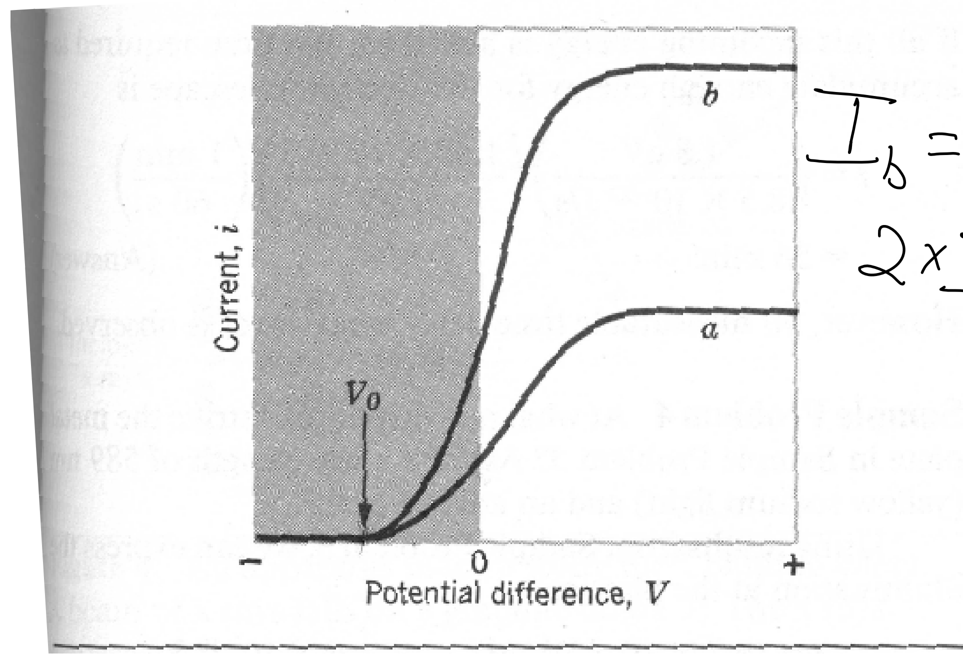


Figure 1 An apparatus used to study the photoelectric effect. The incident light falls on plate *P*, ejecting photoelectrons, which are collected by collector cup *C*. The photoelectrons move in the circuit in a direction opposite to that of the conventional current arrows.

Proof of photons: Photoelectric effect





$I_b = 2 \times I_a$

Stopping Potential

The K.E. of the most energetic electron is measured:

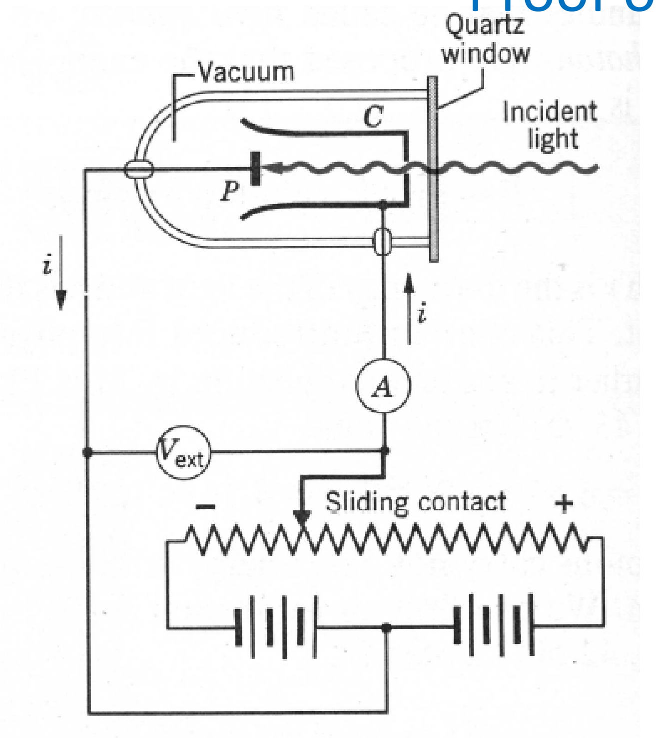
$KE_{max} = eV_0$, independent of intensity (curve a & b)

Classical Expectations:

(1) Energy of ejected electrons $\propto$ Intensity $\propto E^2$

$\Rightarrow$ Expect $KE_{max} \propto$ Intensity

Proof of photons: Photoelectric effect



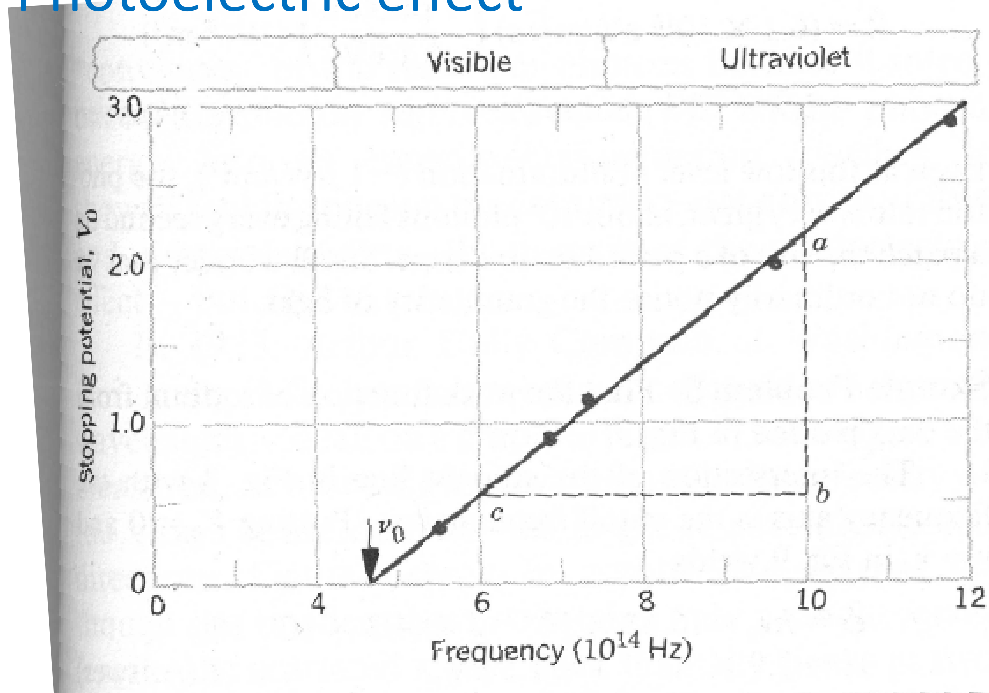


Figure 3 The stopping potential for sodium as a functi

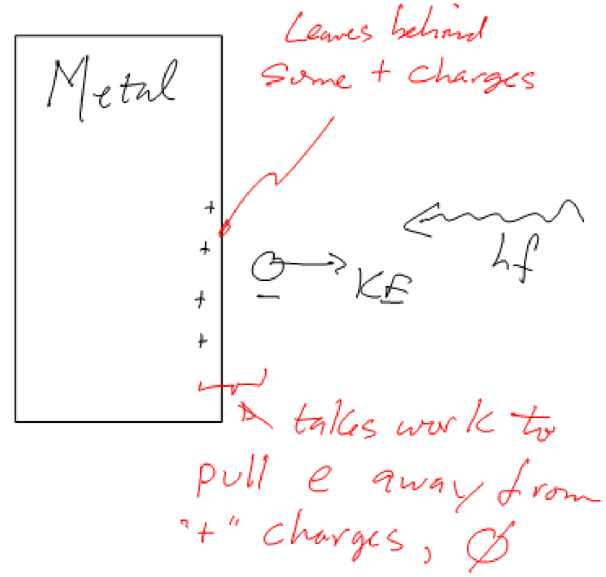
① There exists a cut-off frequency ν_0 below which there is no photoelectric effect

Classical Expectations:

② Photoelectric effect should occur at all frequencies provided

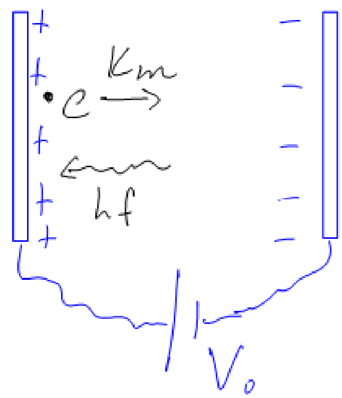
Using Einstein's photon model:

$$\underbrace{hf}_{\text{Photon Energy}} = \underbrace{\phi}_{\text{Work function}} + \underbrace{K_m}_{\text{KE of electron}}$$



or $K_m = hf - \phi$

From circuit analysis, $K_m = eV_0$



if $eV_0 > KE$ of e, it will not make it to the "Right" plate

if $eV_0 = KE$, it will barely make it to the "Right" plate

$\Rightarrow K_m = eV_0$

$$\Rightarrow eV_0 = hf - \phi$$

$$\text{or} \quad V_0 = \underbrace{\left(\frac{h}{e}\right)}_{③}f - \frac{\phi}{e}$$

③ Correct slope measured experimentally

① V_0 linear with f

② Minimum f required to observe photo generated current

Photo electric effect "time delay" issue:

Classically: The Energy of an ejected photo electron must be "soaked up" from the incident wave. The effective area in which the electron soaks up this energy ~ cross sectional area of atom. Therefore, if Intensity is feeble, the will exist a time delay as enough (time integrated) energy is absorbed by an electron

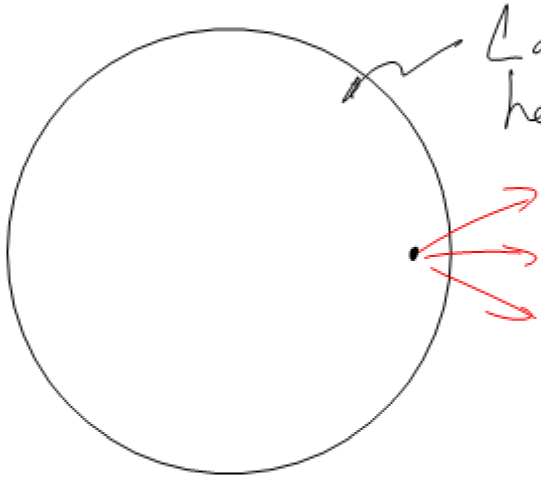
This delay is <u>not</u> <u>measured</u>

Photon Picture: photon energy is delivered to the ejected electron in a single collision event

More proof of quantization of energy: Planck's Black-body radiation

Ideal radiator: a radiator whose emitted radiation depends <u>only</u> on temperature (independent of material, nature of surface)

Make an ideal radiator = black body radiator:



Large hollow closed surface held @ temperture T with small hole

radiation

photons inside are in equilibrium w/walls

$S(\lambda)\,d\lambda$ = radiated Intensity in the interval of wavelength's λ & $\lambda + d\lambda$

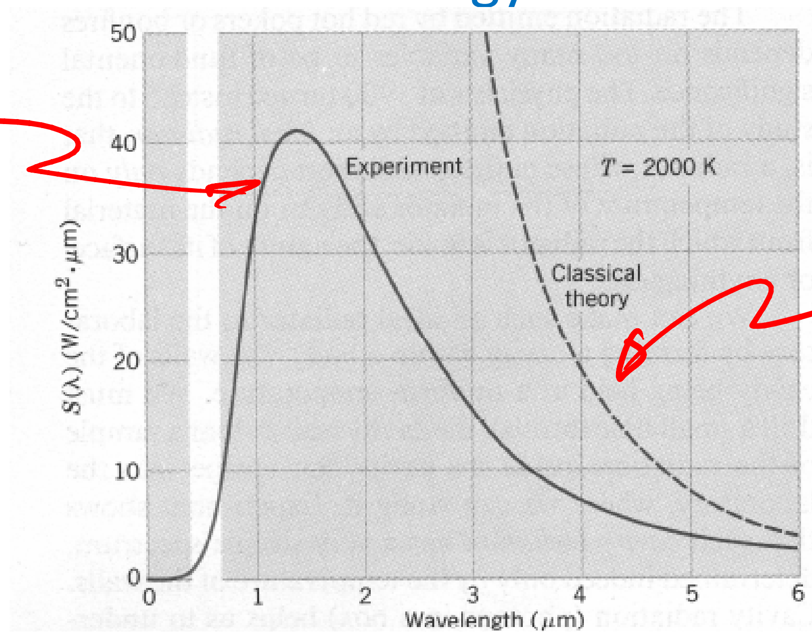
$S(\lambda)$ called Spectral radiance

Classical Law: $S(\lambda) = \frac{2\pi c}{\lambda^4} kT$, k = Boltzmann Constant $= 1.38 \times 10^{-23}$ J/K

More proof of quantization of energy: Planck's Black-body radiation

Experimental Data

$S(\lambda)$



λ

Classical Theory

Classical Law: $S(\lambda) = \dfrac{2\pi c}{\lambda^4} kT$, k = Boltzmann Constant $= 1.38 \times 10^{-23}$ J/K

Under the assumption that the atoms that form the walls of the cavity can exist only in states states of definite (quantized) energy; states with intermediary energies are forbidden:

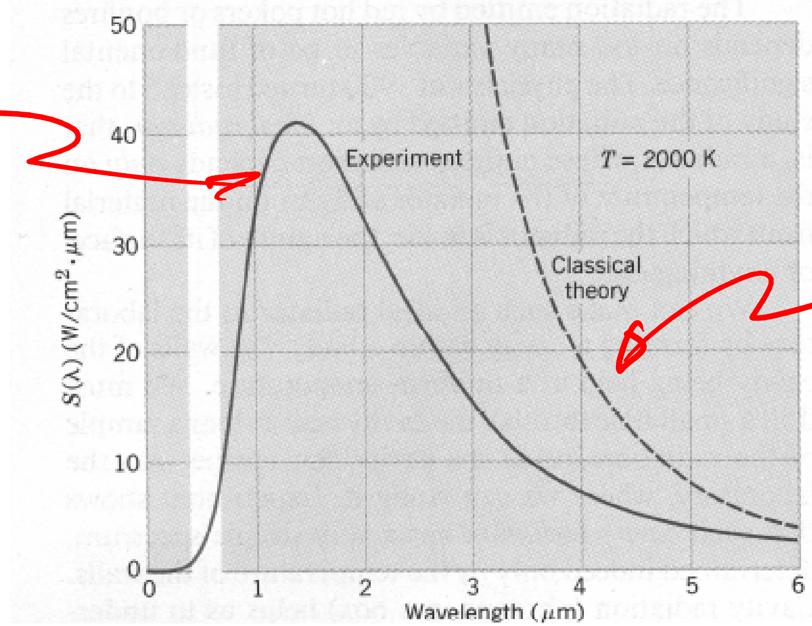
$$S(\lambda) = \frac{2\pi c}{\lambda^4} \frac{hc/\lambda}{e^{\frac{hc/\lambda}{kT}} - 1}$$

Planck's radiation Law

More proof of quantization of energy: Planck's Black-body radiation

Experimental Data

$S(\lambda)$



Classical Theory

λ

Classical Law: $S(\lambda) = \dfrac{2\pi c}{\lambda^4} kT$, k = Boltzmann Constant $= 1.38 \times 10^{-23}$ J/K

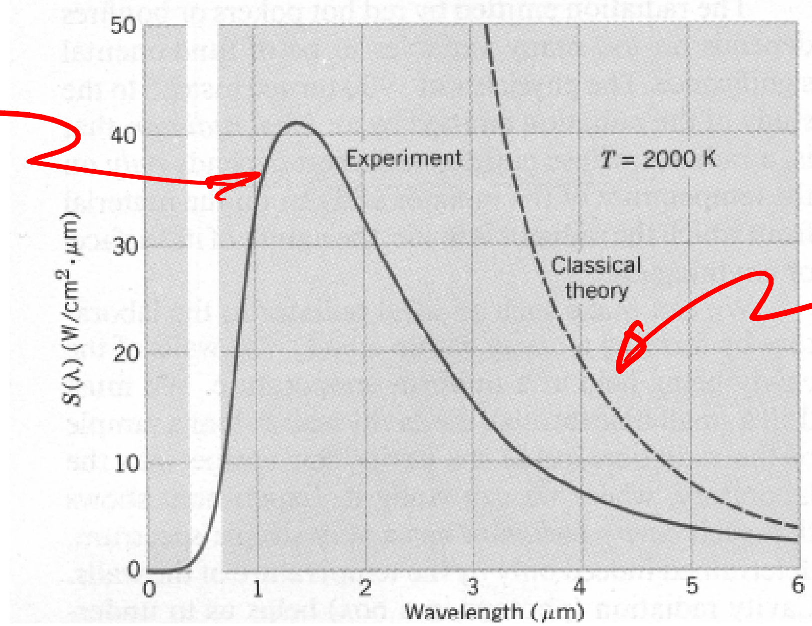
Quantum: $S(\lambda) = \dfrac{2\pi c}{\lambda^4} \dfrac{hc/\lambda}{e^{\frac{hc/\lambda}{kT}} - 1}$ Planck's radiation Law

$$\frac{\partial S}{\partial \lambda} = 0 \Rightarrow \lambda_{peak}(\text{in nm}) = \frac{2.90 \times 10^6 \text{ nm K}}{T}$$

More proof of quantization of energy: Planck's Black-body radiation

Experimental Data

$S(\lambda)$



Classical Theory

λ

Classical Law: $S(\lambda) = \frac{2\pi c}{\lambda^4} kT$, k = Boltzmann Constant $= 1.38 \times 10^{-23}$ J/K

Quantum: $S(\lambda) = \frac{2\pi c}{\lambda^4} \frac{hc/\lambda}{e^{\frac{hc/\lambda}{kT}} - 1}$ Planck's radiation Law

Total Radiated Power per Area $= \int_0^{\infty} S(\lambda)\,d\lambda = \varepsilon \sigma T^4$ (ε = emissivity)

σ = Stefan-Boltzmann Constant $= 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$

De Broglie wavelength for matter – Like Einstein's photon postulate

De Broglie: $p_{ph} = \frac{h}{\lambda}$ Einstein

For matter, postulate: $p = \frac{h}{\lambda} = \gamma_p m u$

Matter wave confined --- particle in a box

In Chapter 21, we found that the wavelength of a standing wave is related to the length L of the confining region by

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, 4, \ldots \tag{25.10}$$

The particle must also satisfy the de Broglie condition $\lambda = h/p$. Equating these two expressions for the wavelength gives

$$\frac{h}{p} = \frac{2L}{n} \tag{25.11}$$

Solving Equation 25.11 for the particle's momentum p, we find

$$p_n = n\left(\frac{h}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots \tag{25.12}$$

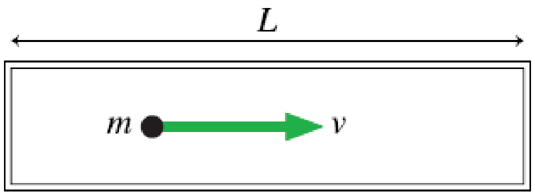
The particle's energy, entirely kinetic energy, is related to its momentum by

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{25.13}$$

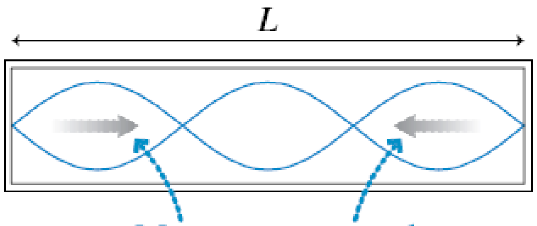
← only for $v \ll c$!

FIGURE 25.16 A particle of mass m confined in a box of length L.

(a) A classical particle of mass m bounces back and forth between the ends.



(b) Matter waves moving in opposite directions create standing waves.



If we use Equation 25.12 for the momentum, we find that the particle's energy is restricted to the discrete values

$$\boxed{E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 \qquad n = 1, 2, 3, 4, \ldots} \tag{25.14}$$

If we use Equation 25.12 for the momentum, we find that the particle's energy is restricted to the discrete values

$$E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 \qquad n = 1, 2, 3, 4, \ldots \tag{25.14}$$

- Only specific discrete energies are allowed: "Quantization" of energy
  "n" is "Quantum number"
  "E_n" is the nth "Energy Level"

- Minimum energy: $E_1 = \dfrac{h^2}{8mL^2}$

the confined particle is <u>always</u> in motion

$$E_n = n^2 E_1$$

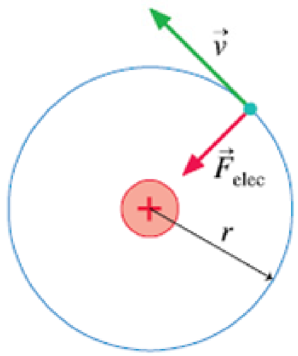
what is waving?!

Bohr model of atom: Semiclassical approach to hydrogen

Bohr Model of Atom Extends Rutherford's model:

1. There exists "stationary states" that have specific quantized energies.
2. The lowest Energy state is "stable" (can not decay). "Transitions" between quantized states occurs by the atom absorbing or emitting the difference in energy between states (includes emitting + absorbing photons)
3. Atoms naturally decay from high energy states to low energy states

FIGURE 39.20 A Rutherford hydrogen atom. The size of the nucleus is greatly exaggerated.



$$F = m\frac{v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \quad ①$$

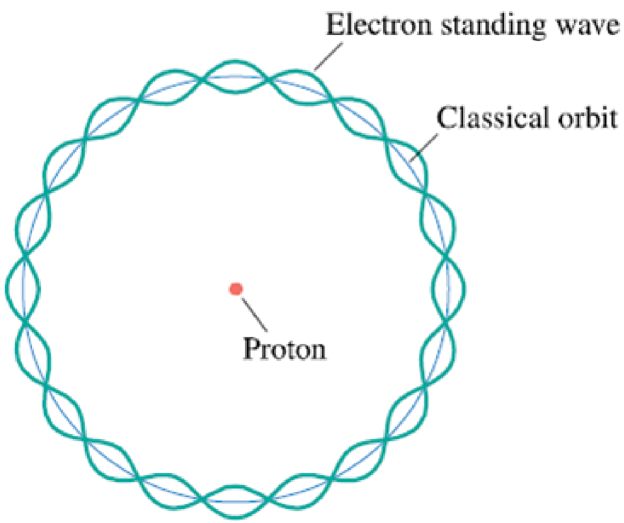
$$\text{\&}\quad p = mv = \frac{h}{\lambda} \quad ②$$

Stationary state:

$$2\pi r = n\lambda \quad ③$$

$$③ \Rightarrow 2\pi r = n\underbrace{\lambda}_{=\frac{h}{mv}\text{ from } ②} = n\frac{h}{mv} \quad ④$$

FIGURE 39.21 An $n = 10$ electron standing wave around the orbit's circumference.



$$\Rightarrow v^2 = \left[\frac{n(h/2\pi)}{mr}\right]^2 = \frac{e^2}{m4\pi\varepsilon_0 r}$$

from ①

$$\Rightarrow r = \left(\frac{n\hbar}{e}\right)^2 \frac{4\pi\varepsilon_0}{m}$$

$$\Rightarrow r_n = n^2 \underbrace{\frac{4\pi\varepsilon_0}{m}\left(\frac{\hbar}{e}\right)^2}_{= a_B = .5\ \text{Å}}$$

$$r_n = n^2 \underbrace{\frac{4\pi\varepsilon_0}{m}\left(\frac{\hbar}{e}\right)^2}_{\equiv a_B = .5\,\text{Å}}$$, the radius of orbits is quantized

From ④ : $2\pi r = n\frac{h}{mv} \Rightarrow v_n = \frac{n}{r_n}\frac{\hbar}{m}$

$$\Rightarrow v_n = \frac{1}{n}\underbrace{\frac{\hbar}{m a_B}}_{v_1 = 2.19\cdot 10^6\ m/s}$$, not relativistic

Total Energy:

$$E = KE + PE = \frac{1}{2}m v_n^2 + \frac{\overbrace{-e^2}^{\frac{e^2}{4\pi\varepsilon_0} = \frac{1}{a_B}\cdot\frac{\hbar^2}{m}}}{4\pi\varepsilon_0 r_n}$$

$$= \frac{1}{2}m\left(\frac{1}{n}\frac{\hbar}{m a_B}\right)^2 - \frac{\hbar^2}{m a_B}\cdot\left(\frac{1}{n^2 a_B}\right)$$

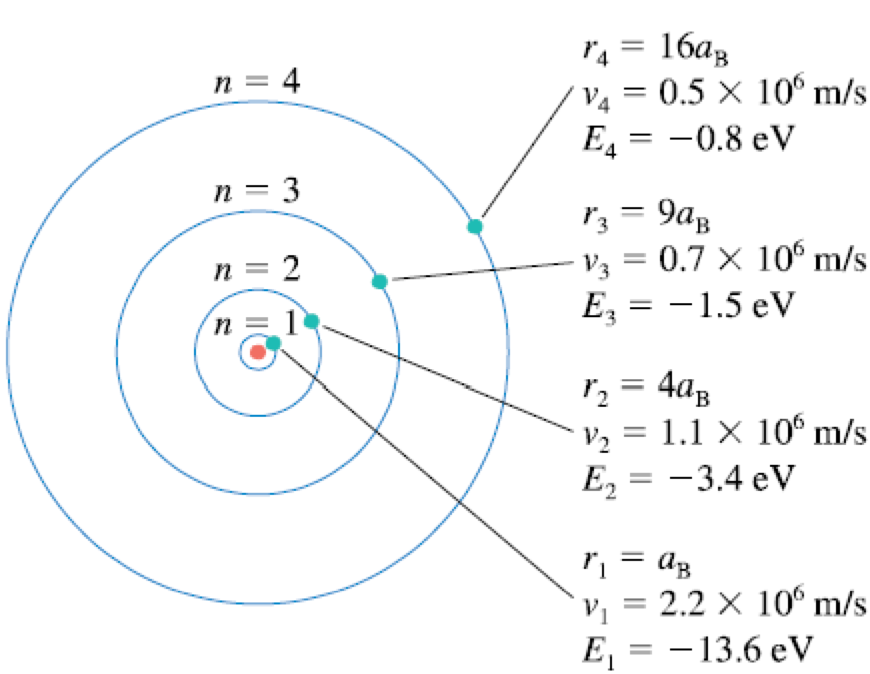
$$= \underbrace{-\frac{m}{2}\left(\frac{\hbar}{m a_B}\right)^2}_{\equiv E_1 = -13.6\ eV}\frac{1}{n^2}$$

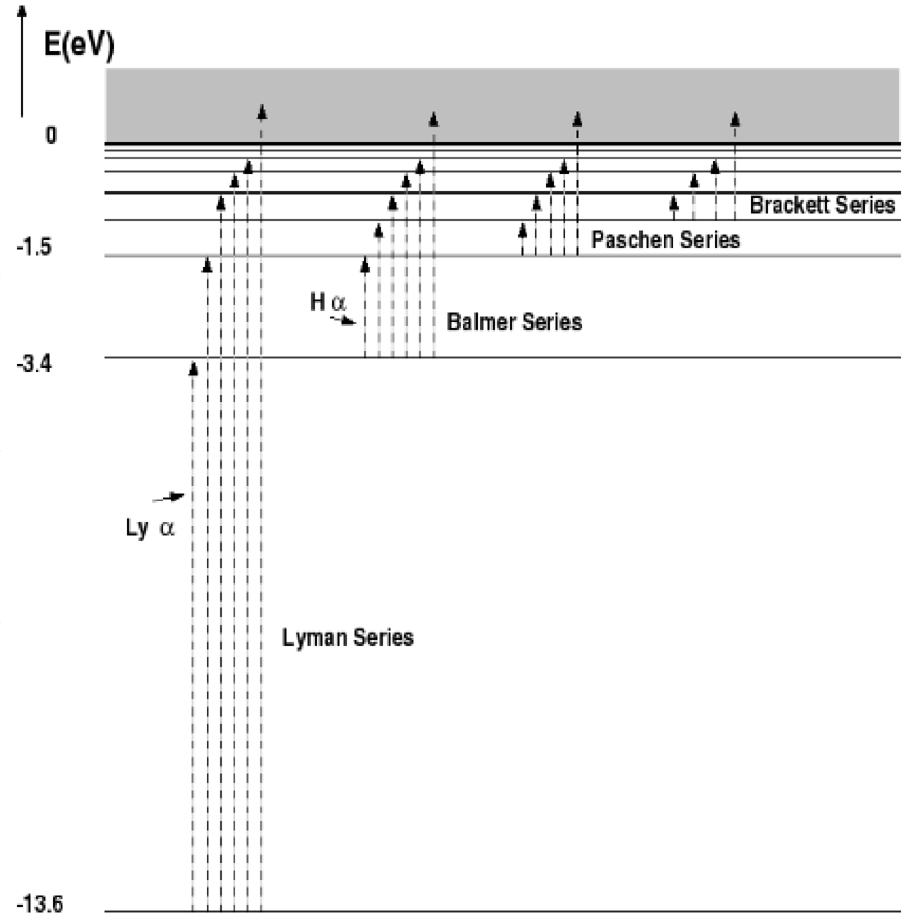
$$\Rightarrow E_n = -\frac{13.60\ eV}{n^2}$$; $1\ Ry \equiv 13.60\ eV$

Rydberg

Bohr model of atom: Semiclassical approach to hydrogen

FIGURE 39.22 The first four stationary states, or allowed orbits, of the Bohr hydrogen atom drawn to scale.





$$E_n = -\frac{Ry}{n^2}$$

Bohr model of atom: Semiclassical approach to hydrogen

Energy Level Diagram

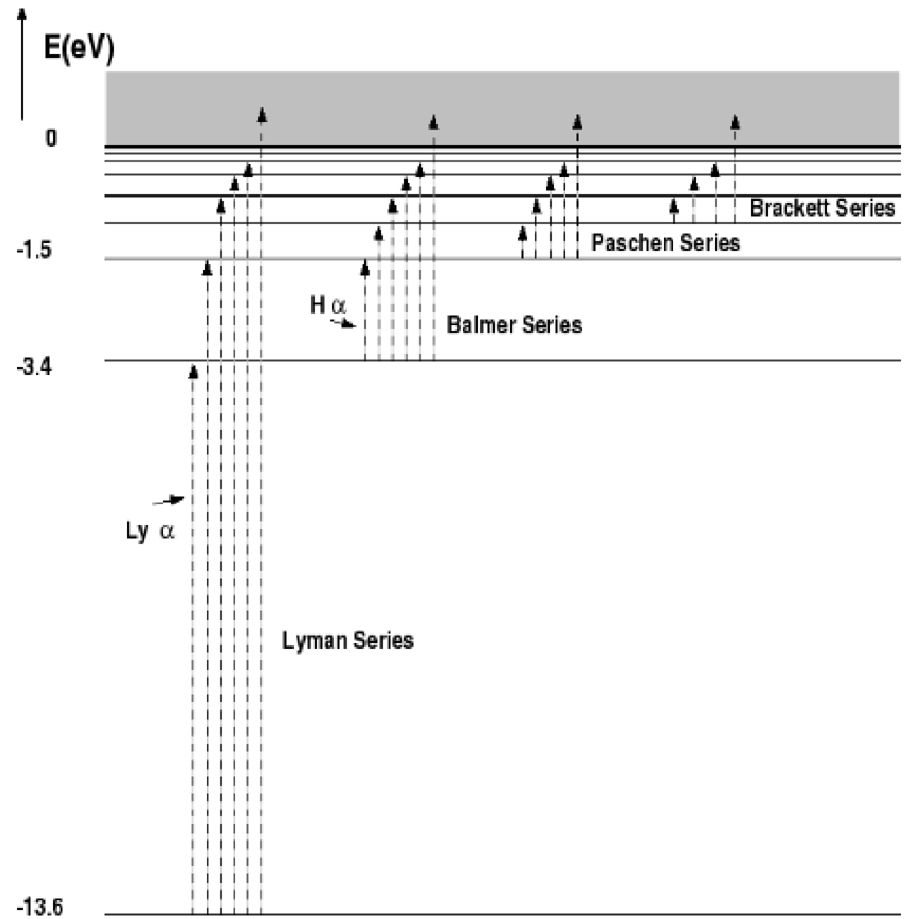
$$E_n = -\frac{R_y}{n^2}$$

$$hf = \Delta E \Rightarrow \frac{hc}{\lambda} = E_m - E_n$$

$$= R_y\left[\frac{1}{m^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow \lambda = \frac{hc/R_y}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} = \left(\frac{\lambda_0}{\frac{1}{m^2} - \frac{1}{n^2}}\right)$$

$$\lambda_0 = 91.18 \text{ nm}$$



absorption & emission transitions

Hydrogen-Like ions: $\frac{e^2}{4\pi\varepsilon_0 r} \rightarrow \frac{Ze^2}{4\pi\varepsilon_0 r}$

or $e^2 \rightarrow Ze^2$

$$a_B = \frac{4\pi\varepsilon_0}{m}\left(\frac{\hbar}{e}\right)^2 \longrightarrow \frac{a_B}{Z}$$

$$r_n = n^2 a_B \longrightarrow r_n = n^2 \frac{a_B}{Z}$$

$$V_n = \frac{1}{n} \underbrace{\frac{\hbar}{m a_B}}_{V_1} \longrightarrow V_n = \frac{1}{n} Z V_1$$

$$E_n = \underbrace{-\frac{1}{2}\left(\frac{\hbar}{m a_B}\right)^2}_{\equiv E_1 = -1\,R_y} \frac{1}{n^2} \longrightarrow E_n = -Z^2 \frac{R_y}{n^2}$$

$$\lambda = \frac{\lambda_0}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \sim \frac{1}{\Delta E} \longrightarrow \lambda = \frac{1}{Z^2} \frac{\lambda_0}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

Bohr model of atom: Semiclassical approach to hydrogen
Quantization of angular momentum

Quantization of angular momentum

Stationary wave condition:

$$2\pi r = n\lambda \quad , \quad \& \ p = \frac{h}{\lambda} = mv$$

$$\Rightarrow 2\pi r = n\frac{h}{mv}$$

$$\Rightarrow \underbrace{mvr}_{\vec{L} = m\vec{v}\times\vec{r} = mvr} = n\hbar$$

$\vec{L} = m\vec{v}\times\vec{r} = mvr$ for circular orbit

$$L_n = n\hbar$$

Probability Density

We can define the probability density $P(x)$ such that

$$\text{Prob}(\text{in } \delta x \text{ at } x) = P(x)\,\delta x$$

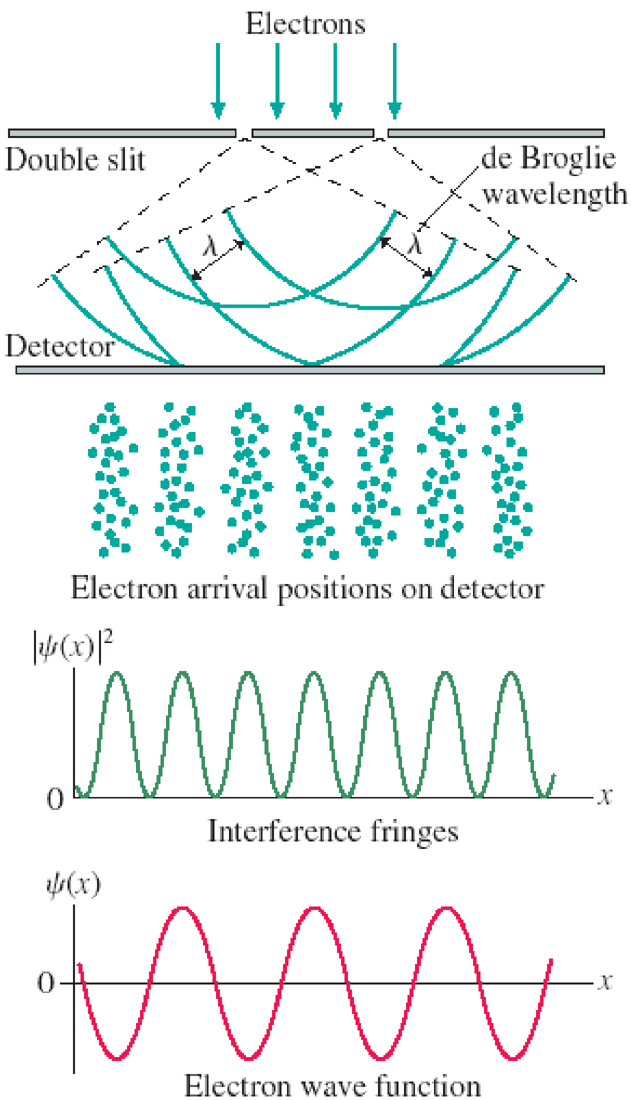
In one dimension, probability density has SI units of m^{-1}. Thus the probability density multiplied by a length yields a dimensionless probability.

NOTE: *P*(*x*) itself is *not* a probability. You must multiply the probability density by a length to find an actual probability.

The photon probability density is directly proportional to the square of the light-wave amplitude:

$$P(x) \propto |A(x)|^2$$

FIGURE 40.5 The double-slit experiment with electrons.



- Electrons behave in similar way: probability density looks just like photons
- Postulate $\exists\ \psi(x)$ for matter waves similar to $A(x)$ for photons — Note that $A(x)$ is the Electric field Amplitude, $\psi(x)$ is something else!
- Probability density $\propto |\psi(x)|^2$ just like photons where $P(x) \propto |A(x)|^2$

Normalization

- A photon or electron has to land *somewhere* on the detector after passing through an experimental apparatus.
- Consequently, the probability that it will be detected at *some* position is 100%.
- The statement that the photon or electron has to land *somewhere* on the *x*-axis is expressed mathematically as

$$\int_{-\infty} P(x)\,dx = \int_{-\infty} |\psi(x)|^2 dx = 1$$

- Any wave function must satisfy this **normalization condition.**

Wave Packets

Suppose a single nonrepeating wave packet of duration Δt is created by the superposition of *many* waves that span a range of frequencies Δf.

Fourier analysis shows that for *any* wave packet

$$\Delta f \Delta t \approx 1$$

We have not given a precise definition of Δt and Δf for a general wave packet.

The quantity Δt is "about how long the wave packet lasts," while Δf is "about the range of frequencies needing to be superimposed to produce this wave packet."

Heisenberg Uncertainty principle:

Wave Packet:

$$\Delta f \, \Delta t \sim 1 \Rightarrow \Delta\omega \, \Delta t \sim 2\pi$$

A Traveling wave spread out in time + frequency is equivalent to being spread out in space (Δx) + spacial frequencies (Δk):

$$\Delta x \, \Delta k \sim 2\pi$$

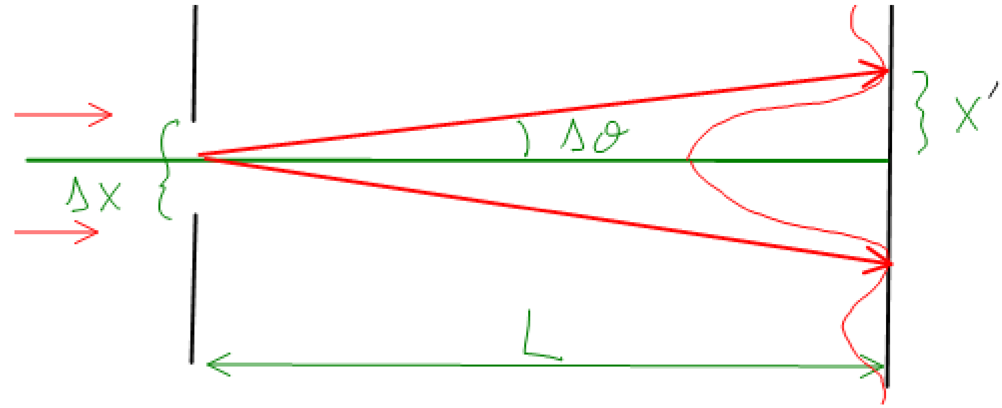
$$+ \quad \Delta k = \frac{2\pi}{\Delta\lambda} \quad ; \quad \frac{1}{\Delta\lambda} = \frac{\Delta p}{h}$$

$$\Rightarrow \Delta x \frac{\Delta p}{\cancel{\hbar}\, h} \sim 1$$

Modern gives $\Delta x \Delta p \geq \frac{\hbar}{2}$

Book gives $\Delta x \Delta p \geq \frac{h}{2}$

$$\text{or} \quad \Delta x \, \Delta p \sim h$$



$$\frac{\Delta x}{2m} \sin \Delta\theta = \frac{\lambda}{2}$$ Single slit, first minimum $m=1$

$$\Rightarrow \sin \Delta\theta = \frac{\lambda}{\Delta x} \simeq \tan \Delta\theta = \frac{x'}{L} \qquad (1)$$

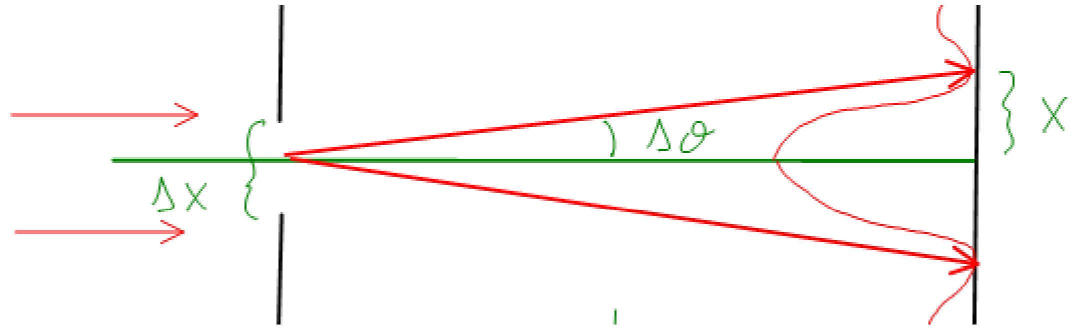
A particle that is "deflected" acquires vertical momentum.
For a particle to land at x requires:

$$x' = V_x t \quad , \quad t = \text{time of flight from slit to screen}$$

$$\simeq \frac{L}{V_0} = \frac{L}{P_0/m}$$

$$V_x = \frac{P_x}{m}$$

$$\Rightarrow x' = \frac{mL}{P_0}\frac{P_x}{m} = \frac{L}{P_0} P_x \quad ; \quad P_0 = \frac{h}{\lambda}$$



$$P_0 = \frac{h}{\lambda} \quad \& \quad x' = \frac{L}{P_0} P_x$$

$$\sin\Delta\theta = \frac{\lambda}{\Delta x} \approx \tan\Delta\theta = \frac{x'}{L} \qquad (1)$$

For a particle to land within Central Maximum:

$$x' = \frac{L\lambda}{h} P_x \Rightarrow \Delta x' = \frac{L\lambda}{h} \Delta P_x$$

$x' \to \Delta x'$
$P_x \to \Delta P_x$

$$(1) \Rightarrow \quad \frac{\lambda}{\Delta x} = \frac{\Delta x'}{L}$$

$$\Rightarrow \quad \Delta x \underbrace{\Delta x'}_{\frac{L\lambda}{h}\Delta P_x} = \lambda L$$

$$\Rightarrow \quad \Delta x \Delta P_x \approx h$$

Slit width

Spread in momentum associated with Central peak