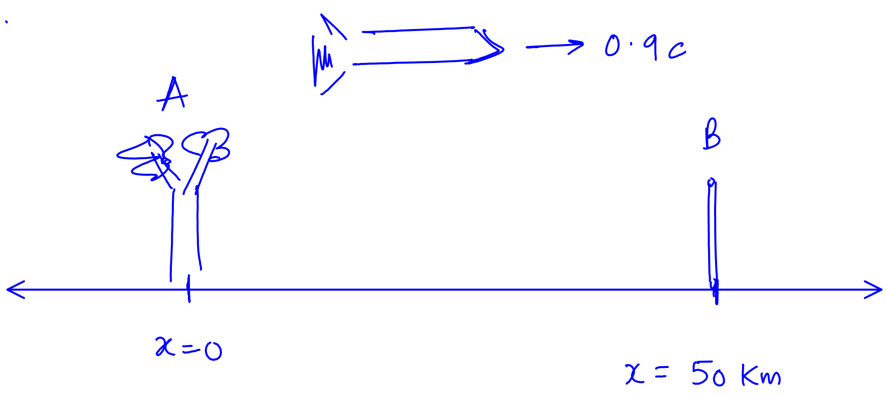
Solution to Problem 1 & 3, Exam III

Wednesday, November 25, 2009
6:17 PM

1.



In the Earth's reference frame, the lightening strikes A and B simultaneously. The coordinates of these "events" are

A: $x = 0$, $t = 5\ \mu sec$ (Earth)

B: $x = 50\ Km$, $t = 5\ \mu sec$ (Earth

The rocket is another reference frame, moving to the right at 0·9c

Their origins coincide at $t=0$

The coordinates in the rocket's frame are related to those in the Earth's frame (For any event A/B)

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad ; \quad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Let the primed coordinates represent rocket frame and the un-primed coordinates represent Earth frame.

(a) For event A:

$$x'_A = \left[0\ Km - 0{\cdot}9 \times 0{\cdot}3\ \frac{Km}{\mu sec} \times 5\,\mu sec\right] \Big/ \sqrt{1 - (0{\cdot}9)^2}$$

$$= \frac{-1{\cdot}35\ Km}{\sqrt{0{\cdot}19}} = -3{\cdot}09\ Km$$

$$t'_A = \frac{5\,\mu sec - 0}{\sqrt{1 - (0{\cdot}9)^2}} = 11{\cdot}47\ \mu sec$$

For event B

For event B

$$x_B' = \left[50\text{ km} - 0.9 \times 0.3 \frac{\text{km}}{\mu\text{sec}} \times 5\ \mu\text{sec}\right] \Big/ \sqrt{0.19}$$

$$= \frac{48.65}{\sqrt{0.19}} \text{ km} = 111.61 \text{ km}$$

$$t_B' = \left[5\ \mu\text{sec} - \frac{0.9 \times 0.3 \frac{\text{km}}{\mu\text{sec}}}{\left(0.3 \frac{\text{km}}{\mu\text{sec}}\right)^2} \times 50\text{ km}\right] \Big/ \sqrt{0.19}$$

$$= -145/\sqrt{0.19}\ \mu\text{sec} = -332.65\ \mu\text{sec}$$

(b) Clearly, event B occured first in the rocket frame, since $t_B' < t_A'$

$$\Delta t' = t_A' - t_B' = 11.47 - (-332.65)\ \mu\text{sec}$$

$$= 344.12\ \mu\text{sec}$$

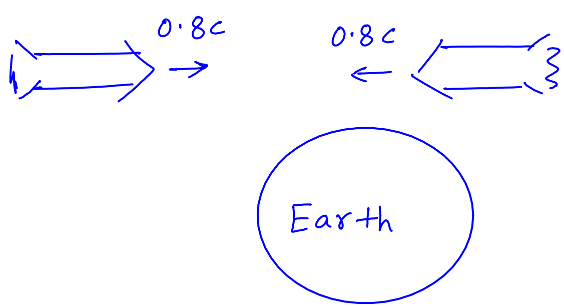
So, the lightening strikes the pole first.

(c)

$$\Delta x' = x_B' - x_A' = 111.61 - (-3.09) \text{ km}$$

$$= 114.7 \text{ km}$$

3.



Length of each rocket, measured in its rest frame is 200 m.

(a) Speed of rocket A, as measured by B is

$$\frac{0.8 + 0.8}{1 + (0.8)^2} = \frac{1.6}{1.64} = 0.975c$$

(b) rocket A has a rest length of 200 m.
In another inertial frame, that of rocket B, which moves at a velocity $v = 0.975\,c$, the length of rocket A would appear as

$$L = \sqrt{1 - \beta^2}\; l$$

$$= \sqrt{1-(0.975)^2} \times 200 = 0.22 \times 200 \text{ m}$$

$$= 44 \text{ m}$$