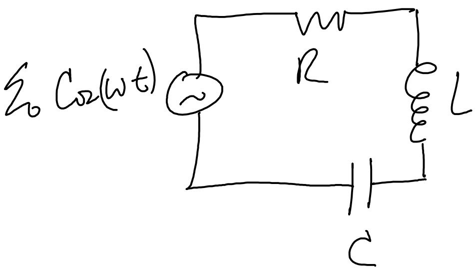
Exam I solutions

Wednesday, September 30, 2009
7:25 PM

5. [10 pts] The resonant frequency of a series RLC circuit driven by an oscillating voltage, $\varepsilon = \varepsilon_o \cos(\omega t)$, is greater than the driving frequency ω. Does the current lead or lag the emf? Your answer will be graded on your explanation which should include a sketch of the circuit, a phasor diagram of the circuit, and a comparison of the magnitude of V_L and V_C, the peak voltages across the inductor and capacitor.

This was a homework problem 36.8 + strongly related to 36.7



Resonant frequency $\Rightarrow$ $V_L = V_C$

or $I_o X_L = I_o X_C$

$\Rightarrow$ $X_L = X_C$

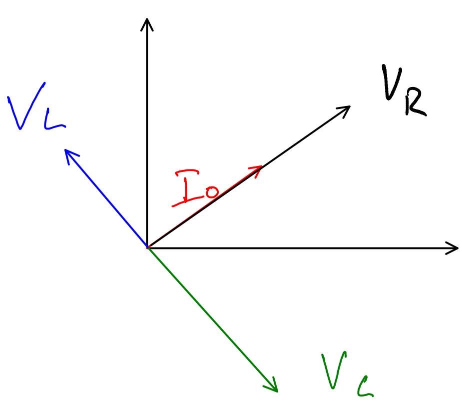
where $X_L = \omega L$ + $X_C = \frac{1}{\omega C}$

for driving frequencies $\omega < \omega_{res}$

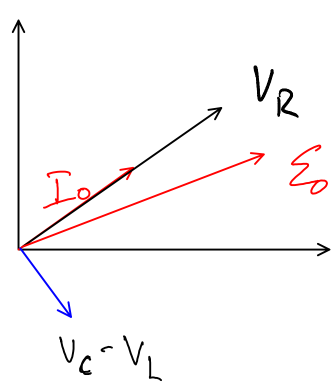
$\Rightarrow$ $X_L < X_C$

Since X_L decreases + X_C increases as freq is reduced from ω_{res}

$\Rightarrow$ $V_L < V_C$



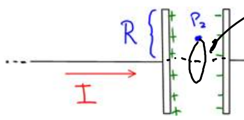
$\Rightarrow$



$\therefore$ Current leads the applied voltage

6. [15 pts] A long wire carrying a current I is charging a large circular parallel plate capacitor with radius R as shown below. Derive an expression for the magnitude of the magnetic field midway between the parallel plate capacitors at a distance r from the center where r<R (at point P2).

Quiz #3d, HWK 35.38



Contour is circle, centered on wire & ∥ to plates, radius r

$$\oint B \cdot ds = \mu_0\left(I_{enc} + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$

$I_{enc} \to 0$: No current through loop

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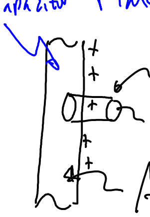
E-field for a Capacitor:



choose a Gaussian pill-box with area $A_\perp$

A_{plate}, area of capacitor plate

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow E A_\perp = \frac{\left(Q \frac{A_\perp}{A}\right)}{\varepsilon_0}$$ where Q is total charge on plate

$$\Rightarrow E = \frac{Q/A}{\varepsilon_0} = \frac{Q}{(\pi R^2)\varepsilon_0}$$

Now, $\Phi_E = \oint E \cdot da = E \pi r^2 = \frac{Q}{\varepsilon_0} \frac{r^2}{R^2}$

$$\& \frac{d\Phi_E}{dt} = \frac{dQ}{dt} \frac{1}{\varepsilon_0} \frac{r^2}{R^2} = \frac{I}{\varepsilon_0} \frac{r^2}{R^2}$$

$$\therefore \oint B \cdot ds = B \cdot 2\pi r = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\Rightarrow B = \frac{\mu_0 \varepsilon_0}{2\pi r} \frac{I}{\varepsilon_0} \frac{r^2}{R^2}$$

$$\Rightarrow B = \frac{\mu_0}{2\pi} \frac{r}{R^2} I$$

Derived in class & in Book (eq 35.24) waves propagate along $\vec{S} \sim \vec{E} \times \vec{B}$; $E_0/c = B_0$; $\vec{S}$ is intensity; meaning of "plane wave";

4. [15 pts] An electromagnetic plane wave with wavelength λ and period T travels in the +z-direction in vacuum where the electric field is polarized in the x-direction and has an amplitude Eo.

a. [3 pts] Write a vector expression of the electric field for arbitrary time t and position z.

$$\vec{E} = E_0 \cos(kz - \omega t)\hat{x}, \quad k = \frac{2\pi}{\lambda}, \quad \omega = \frac{2\pi}{T}$$

b. [3 pts]Write a vector expression of the magnetic field for arbitrary time t and position z.

$\hat{x} \times +\hat{y} = \hat{z}$, propagation direction: $\vec{S} \propto \vec{E} \times \vec{B}$

$$\vec{B} = B_0 \cos(kz - \omega t)(+\hat{y}), \quad B_0 = \frac{E_0}{c}, \quad k = \frac{2\pi}{\lambda}, \quad \omega = \frac{2\pi}{T}$$

c. [4 pts] What is the magnitude and direction of the electric field vector at position x=0.3 λ, y=0.9 λ , and z= 0.1 λ at time t=0.3 T?

$$\vec{E} = E_0 \cos\left(\frac{2\pi}{\lambda}(.1\lambda) - \frac{2\pi}{T}(.3T)\right)\hat{x} = E_0 \underbrace{\cos[2\pi(-.2)]}_{.31}\hat{x}$$

$= (\cdot 71) E_0 \; X$

d. [5 pts] Assume that the above wave is incident on a perfectly absorbing circular plate of radius R oriented parallel to the x-y plane. Derive an expression for the total energy absorbed over a time interval equal to 10 periods, t=10.0 T.

$$S_{avg} = \frac{1}{2\mu_0} \vec{E}_0 \times \vec{B}_0 = \frac{\text{Energy/Area}}{\text{time}} = \frac{\text{Energy}}{(\pi R^2 \cdot 10T)}$$

$$\Rightarrow \text{Energy} = \left[\frac{1}{2\mu_0} E_0 \frac{E_0}{c}\right] \pi R^2 \, 10T \quad , \; c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$= 5T \sqrt{\frac{\varepsilon_0}{\mu_0}} \pi R^2 E_0^2$$

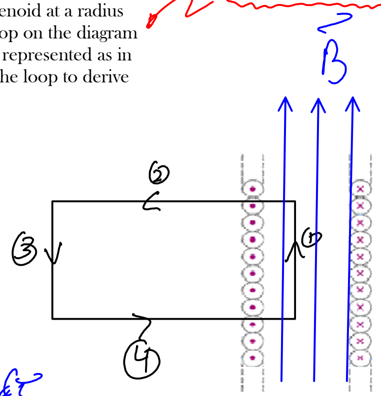
Entire problem similar to HwK #34.40

Derivation in class + in Book; one of the few examples of utility of Ampere's Law

3. [30 pts] A solenoid with N turns, length L, and radius R has a current

I= I() (1+ t/τ) where τ and I() are both positive constants and t is time.

a. [8 pts] Derive an expression for the magnetic field inside the solenoid at a radius of r where r<R at time t=0. Make sure you draw your Amperian loop on the diagram (which depicts a cross sectional view of a solenoid with the current represented as in and out of the page) and describe in detail the integration around the loop to derive the B-field.



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$\int_{①} \vec{B} \cdot d\vec{s} + \int_{②} \vec{B} \cdot d\vec{s} + \int_{③} \vec{B} \cdot d\vec{s} + \oint_{④} B \cdot d\vec{s}$$

Symmetry of infinite cylinder

$\Rightarrow$ B is constant along ①

@ infinity, the currents appear to cancel $\Rightarrow B \to 0$

$\therefore$ if leg ③ were @ infinity, then $\int_{③} \vec{B} \cdot d\vec{s} = 0$

Since all loops outside of solenoid

have no current going through them $\Rightarrow \int_{③} \vec{B} \cdot d\vec{s} = 0$ everywhere

Leg ② + ④ give zero

inside solenoid since $\vec{B} \perp d\vec{s}$

outside solenoid since $B = 0$

$$\therefore \int \vec{B} \cdot d\vec{s} = \int_{①} B \, ds = B\ell$$

$$I_{enc} = I_0\left(\frac{N}{L}\ell\right)$$

(I_0: current through wire)

$$\therefore \quad B\ell = \mu_0 I_0 \frac{N}{L}\ell \Rightarrow B = \mu_0 \frac{N}{L} I_0$$

b. [2 pts] Draw the direction of the magnetic field inside the solenoid at time t=0 on the diagram above which depicts a cross sectional view of the solenoid.

→ RHR from many Homework Problems

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c. [15 pts] What is the magnitude of the electric field at radius r<R inside the same solenoid at some arbitrary time t>0?

→ Faraday's law: Quiz #3a

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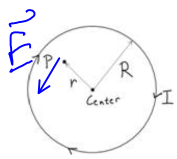
$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \Rightarrow E\cdot 2\pi r = \pi r^2 \frac{dB}{dt}$$

$$B = \mu_0 \frac{N}{L} I_0 (1 + t/\tau) \Rightarrow \frac{dB}{dt} = \mu_0 \frac{N}{L} I_0 \frac{1}{\tau}$$

$$\therefore \quad E = \frac{r}{2}\mu_0 \frac{N}{L}\frac{I_0}{\tau}$$

d. [5 pts] Draw the direction of the electric field at point P (at radius r) on the diagram below depicting a bottom view of a solenoid with the current moving clockwise. Explain your reasoning.



Lenz's Law: lots of ex's

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Flux is increasing into page

Lenz's law ⇒ $\vec{E}$-field produced which, if a hypothetical conducting loop were placed in the solenoid, would drive a current CCW to oppose the increasing flux ⇒ $\vec{E}$ pts CCW

2. [20 pts] A current I0 is present in the two circularly arced wire segments of radius R depicted below as thick black lines. One arc subtends an angle of π/2, and the other subtends and angle of π/4. The lead wires point radially outward from point P, the location of the center of the two arced wire segments. Pay special attention to the direction of the current in the two arcs. Derive from the Biot-Savart law the magnitude and direction of the magnetic field at the point P.

B-field from Arc: Example in class + homework problem 33.46 + Quiz #1a + #1d

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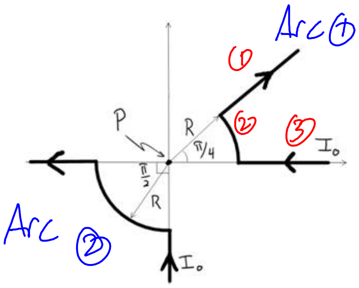

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$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{s}\times\hat{r}}{r^2}$$

Arc ①

Leg ① + ③ : $d\vec{s} \parallel \hat{r} \Rightarrow d\vec{s}\times\hat{r} = 0$

$$\therefore \int d\vec{B} = \int_{②} \frac{\mu_0}{4\pi} I \frac{d\vec{s}\times\hat{r}}{r^2}, \quad d\vec{s} \perp \hat{r},\ ds = R\,d\theta,\ r = R$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} I \frac{1}{R}\int d\theta = \frac{\mu_0}{4\pi}\frac{I}{R}\frac{\pi}{4} \text{ out of page}$$

Arc 2: $\vec{B} = \frac{\mu_0}{4\pi} I \frac{1}{R}\left(\frac{\pi}{2}\right)$ into page

$$\therefore \vec{B}_{total} = \vec{B}_{①} + \vec{B}_{②} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{\mu_0}{4\pi}\frac{I}{R}\frac{\pi}{4} \text{ into page}$$

1. [10 pts] Laboratory scientists have created the electric and magnetic fields shown below: the electric field points downward and to the right with magnitude 1.0 x 10^6 V/m, and the magnetic field is into the page with magnitude 1.0 T. These fields are also observed by scientists that zoom through these fields in a rocket traveling in the +x-direction at a speed of 1.0 x 10^6 m/s. [For the pedantic reader, assume the rocket is made of insulating material.]

Quiz #3b, Hmwk 35.5

a. [5 pts] According to the scientists inside the rocket, what is the magnetic field expressed as a vector in x, y, and z components?

$$\vec{B}' = \vec{B} - \frac{\vec{V}}{c^2}\times\vec{E} = 1.0\,T(-\hat{z}) - \frac{10^6}{(3\cdot 10^8)^2} 10^6\, T \underbrace{\cos 45°}_{\sqrt{2}/2} (-\hat{z})$$

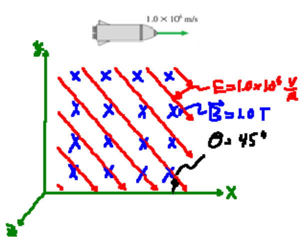
$$= -\left[1.0\,T + \left(\frac{10^{-4}}{9}\frac{\sqrt{2}}{2}\right)T\right]\hat{z} \approx +1.0\,T(-\hat{z})$$

b. [5 pts] According to the scientists inside the rocket, what is the electric field expressed as a vector in x, y, and z components?

$$\vec{E}' = \vec{E} + \vec{V}\times\vec{B}$$

$$= 10^6\left(\frac{\sqrt{2}}{2}\hat{x} - \frac{\sqrt{2}}{2}\hat{y}\right) + 10^6(1.0)\hat{y}$$

$$= 10^6\left(\frac{\sqrt{2}}{2}\hat{x} + \left(1 - \frac{\sqrt{2}}{2}\right)\hat{y}\right)\frac{V}{m}$$



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