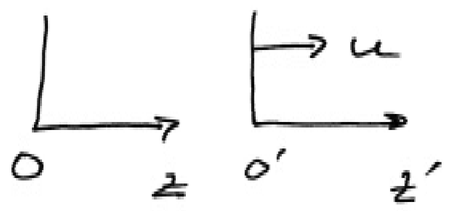
Relativity Review

To know:

- Inertial Reference Frames
  - ⇒ physics is the same
- Lorentz transforms
  - Position and time (Contraction / Dilation)
  - velocity note: $v_{\parallel}$ & $v_{\perp}$ not the same.
- Relativistic Doppler Shift
- Energy & Momentum
- photons & equivalence principle

Lorentz transformations



$$x' = x \; ; \; y' = y$$

$$z' = \gamma(z - ut) \; ; \; t' = \gamma\left(t - \beta\frac{z}{c}\right)$$

Where $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$; $\beta = \dfrac{u}{c}$

For the inverse transforms $x' \rightarrow x$

let $u \rightarrow -u$ and switch ' and un'

For the velocities

$$v_x' = \frac{v_x}{\gamma\left(1 - \beta\frac{v_z}{c}\right)} \; ; \; v_y' = \frac{v_y}{\gamma\left(1 - \beta\frac{v_z}{c}\right)}$$

$$v_z' = \frac{v_z - u}{1 - \beta\frac{v_z}{c}}$$

Again to go from ' to un', switch ' and un' and let $u \rightarrow -u$

Some Consequences

- Lorentz Contraction $\qquad L_{moving} = \frac{L_{rest}}{\gamma}$

- Time Dilation $\qquad \Delta t_{moving} = \gamma \, \Delta t_{rest}$

- Doppler Shift

$$\omega_{moving} = \omega_{rest} \sqrt{\frac{1 \pm \beta}{1 \mp \beta}}$$

where upper sign approaching
low sign receding

$$\lambda_{moving} = \lambda_{rest} \sqrt{\frac{1 \mp \beta}{1 \pm \beta}}$$

Consequence Continued

- Relativistic Energy
  - Rest energy of massive particle

$$mc^2$$

  - Total Energy of massive particle

$$\gamma mc^2$$

  - Kinetic Energy of massive particle

$$\gamma mc^2 - mc^2 = mc^2(\gamma - 1)$$

- Relativistic Momentum
  - Massive particle

    $\gamma m v$

- Energy & Momentum for Massless particle like the photon

$$\mathcal{E} = pc = h\nu = \frac{hc}{\lambda}$$

total energy!

$$\Rightarrow \quad p = \frac{h}{\lambda}$$

where h = Planck's Constant

For massive particle we have another expression for the total Energy

$$\mathcal{E} = \sqrt{\underbrace{m^2c^4} + p^2c^2}$$

must add the rest energy to the energy due to its motion

Quantum

To know:

- Bohr Atom — 1-electron atom
- de Broglie wavelength
- Uncertainty principle

Wave nature of matter

- Schrödinger Equation (time-independent)
- Wave function, probability density
- Example of infinite square well potential

The Photon

Quantum Mechanics (Photoelectric Effect, Compton Scattering, etc.) tell us light comes in packages of energy of quantity $\underline{\underline{h\nu}}$. How do we reconcile this with our notion of intensity in a beam of light?

$$I = \frac{\text{Power}}{\text{unit area}} = \frac{\text{Energy/unit time}}{\text{unit area}}$$

$\Rightarrow$ introduce the notion of

Photon flux, η,

photon per unit time per unit area

$$I = \eta h\nu = \frac{n}{\Delta t \cdot A} h\nu$$

Δt = length of time

A = area

total Energy in the beam

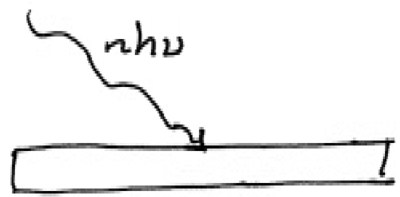
$$\mathcal{E} = I \cdot \Delta t \cdot A = n h\nu$$

amount of energy in each photon package

# of photons

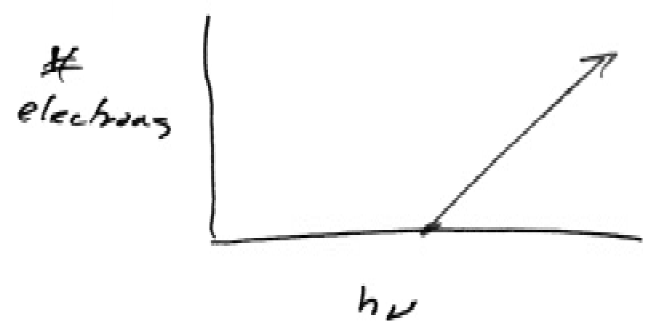
$\Rightarrow$ there are two aspects to the energy of light – the amount of energy in each unit & the number of units.

This means



If we increase the energy by raising the energy (and thus the intensity) by increasing the # of photon no electrons are liberated if $h\nu <$ Work function

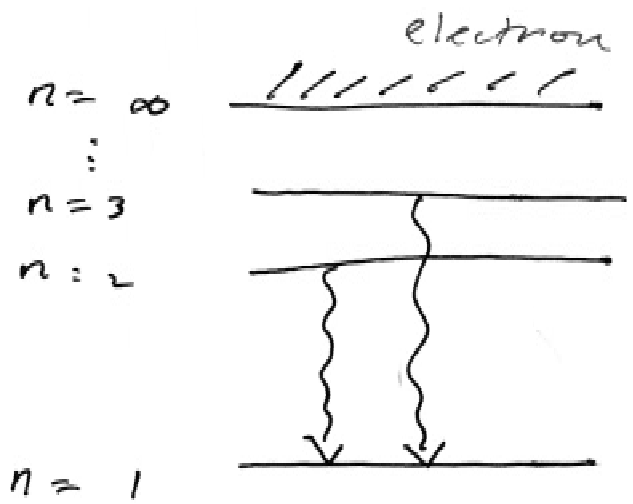
If we increase the energy by raising ν, we will eventually reach a threshold where electrons are liberated.



Quantum Mechanics

$\underbrace{h \text{ \& } \hbar = \frac{h}{2\pi}}_{\text{has units of energy-time}}$ should know in units of J-s & eV-s

1- electron atom Rydberg formula tells us the wavelength of the light due to transitions between bound (stationary) energy levels



$$\frac{1}{\lambda_{nn'}} = R_\infty \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)$$

Note $\frac{1}{\lambda_{n\infty}} = \frac{R_\infty}{n^2}$

so for $n=1$ $\lambda_{n\infty} = \frac{1}{R_\infty}$

In the Bohr model

$$E_n = -\frac{1}{2}\left(\frac{e^2}{4\pi\epsilon_0}\right)\frac{1}{n^2 a_0}$$ for the levels of hydrogen

$$= -hc\frac{R_\infty}{n^2}$$ a_0 = Bohr radius

$$\underbrace{\Delta E = E_{n'} - E_n} = \frac{hc}{\lambda_{nn'}} = hcR_\infty\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$$

the amount of energy required to excite an electron for level n to a higher level n'

Note, the larger ΔE is the smaller $\lambda_{nn'}$ is!

Wave nature of Matter.

As with light de Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{non relativistically}$$

w/ this all interference & diffraction you learned for light now applies to matter.

As a result of the wave nature we have the uncertainty principle

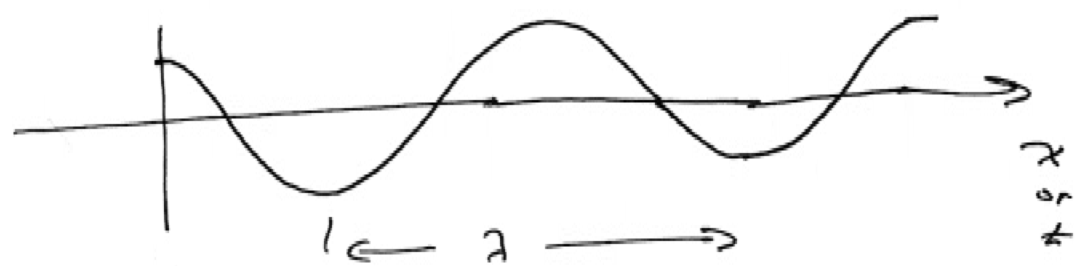
$$\Delta x \, \Delta p \geq \frac{\hbar}{2} \quad \text{and} \quad \Delta E \, \Delta t \geq \frac{\hbar}{2}$$

What does this mean?

You cannot deterime both the position and the momentum to arbitrary accuracy. Consider a wave of a single frequency.

$$\psi(x,t) = A\cos(kx - \omega t)$$

This describes a plane wave. We know very well k & ω but as a consequence the particle is everywhere at the same time



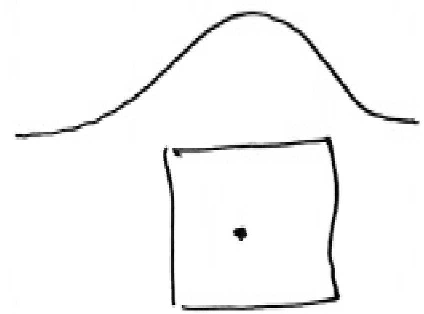
λ is know exactly $\Rightarrow$ k is known exactly

$p = \hbar k \Rightarrow p$ known exactly

but x runs from $-\infty$ to $+\infty$

we have no knowledge of where the particle is!

Now say



L

$= \sum_{k_i} \cos(\underline{k_i} x)$

different frequencies!

we know where the particle is to $\Delta x = L$

in order to restrict the wave function to this box we had to add waves of different frequencies

So $\Delta p \simeq \frac{\hbar}{2\Delta x} \simeq \frac{\hbar}{2L}$

Time-independent Schrödinger Equation

$$\frac{d^2}{dx^2}\psi(x) = \frac{2m}{\hbar^2}\left[V(x) - E\right]\psi(x)$$

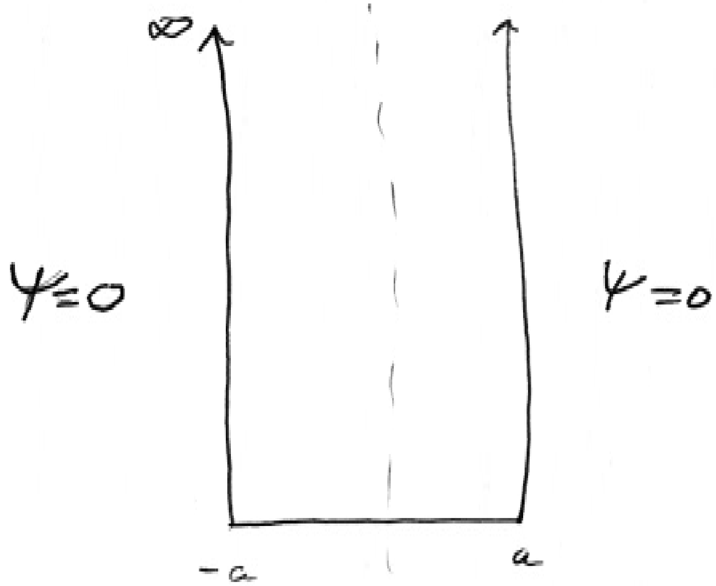
→ Solve for $\psi(x)$ for given $V(x)$

When $V(x) = 0$ $\quad \frac{d^2}{dx^2}\psi(x) = -\frac{2mE}{\hbar^2}\psi(x)$

Solution $\psi(x) = A\cos kx + B\sin kx$

Where $k = \frac{\sqrt{2mE}}{\hbar}$

Example particle in infinite square well potential



inside $\psi(x) = A\cos kx + B\sin kx$

$\psi(a) = \psi(-a) = 0$

Even solutions

$A\cos ka = 0 \Leftarrow$ Cos an even function!

$ka = \frac{n\pi}{2} \quad n = 1, 3, 5 \ldots$

$k = \frac{n\pi}{2a}$

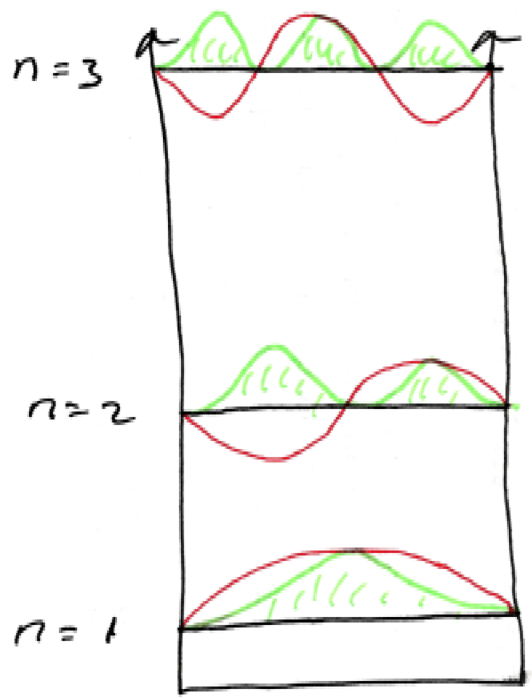
odd solutions

$B\sin ka = 0$

$k = \frac{n\pi}{2a} \quad n = 2, 4, 6, \ldots$

Kinetic Energy is now quantized

$$\mathcal{E}_n = \frac{\hbar^2 k_n^2}{2m} = n^2 \frac{\pi^2 \hbar^2}{8ma^2} = \frac{n^2 h^2}{32 m a^2}$$



$\psi(x)$ the amplitude

$|\psi(x)|^2$ the probability density

$$\text{Prob} = \int_{-a}^{0} |\psi(x)|^2 \, dx$$

$$= 50\%$$

Probability for finding the particle between $-a$ and 0.