Exam I Review

- Lorentz Force:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- Magnitude of B force:

$$q|\vec{v} \times \vec{B}| = q|\vec{v}||\vec{B}|\sin\theta$$

θ = angle between $\vec{v}$ & $\vec{B}$

- Dimensions of B

$$T \equiv \frac{N}{A \cdot m} \quad \text{Tesla}$$

$$G = 10^{-4}\,T \quad \text{Gauss}$$

- When $\vec{v} \perp \vec{B}$

Charge moves in a circle

$$r = \frac{mv}{qB}$$

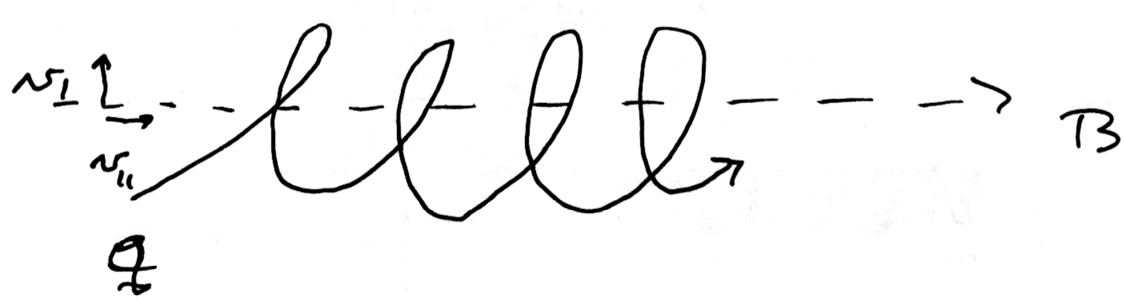
cyclotron frequency

$$\nu = \frac{qB}{2\pi m}$$

note: independent of v & r!

- When $\vec{v}$ has tangential component

Charge moves in a spiral



- B-field does no work

$$\vec{F} \cdot \vec{v} = 0 \quad \text{since } \vec{F} \perp \vec{v}$$

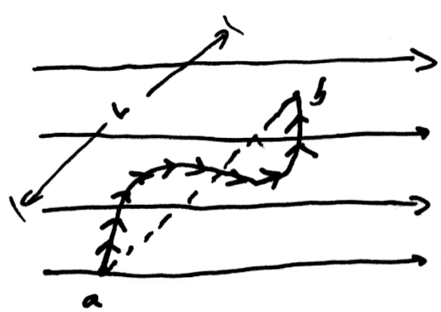
$$\Rightarrow \frac{du}{dt} = 0$$ does not change the energy!

- Force on a wire segment

$$\vec{F}_{mag} = I \underbrace{\int_a^b d\vec{s} \times \vec{B}}_{}$$

note: this is a vector sum

$$= I \vec{L} \times \vec{B}$$



- Force on a closed loop in a uniform B-field

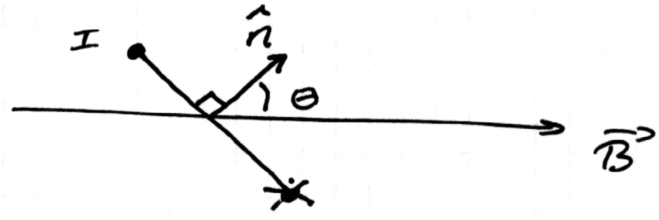
$$\vec{F}_{mag} = I \oint d\vec{s} \times \vec{B} = 0$$

- Closed loop can feel a torque

$$\vec{\tau}_{mag} = I A B \sin\theta \, \hat{\theta}$$

A = area of the loop

sign of the torque determined by $\hat{n} \times \vec{B}$



$\hat{n}$ normal to surface $\parallel \vec{A}$

define a magnetic moment

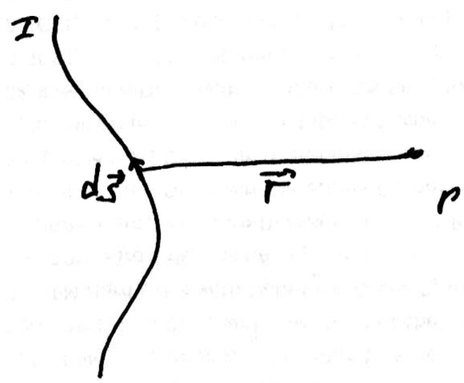
$$\vec{\mu} = I\vec{A}$$ w/ units A-m^2

- When B is not uniform

$$\oint d\vec{s} \times \vec{B} \neq 0 \quad !$$

- Biot - Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{s} \times \hat{r}}{r^2}$$



$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{s} \times \hat{r}}{r^2}$$

- Ampère's Law

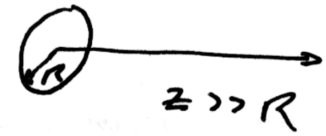
$$\oint \vec{B} \cdot d\vec{l} = \underbrace{\mu_0 I}_{\text{Current due to wires}} + \mu_0 \epsilon_0 \underbrace{\frac{d\phi_E}{dt}}_{\text{Change in electric flux}}$$

$\mu_0 = 4\pi \times 10^{-7}$ $\underbrace{T \cdot m/A}$ also $N/A \cdot m$ Vacuum permeability

$\epsilon_0 = 8.854 \times 10^{-12}\ C^2/N \cdot m^2$ Vacuum permittivity

$\frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$ speed of light in vacuum.

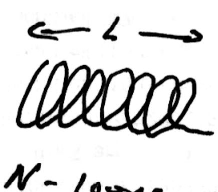
- Some key Magnetic Fields

Current loop 

for field

$B = \frac{\mu_0}{2\pi} \frac{IA}{z^3}$

on-axis



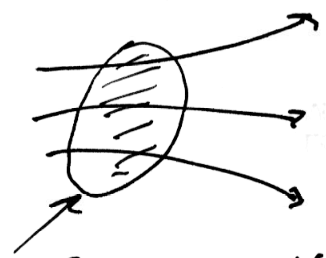
Solenoid

$B = \mu_0 \frac{N}{L} I$

Technically true for very long solenoid near the center.

- Electric and Magnetic Flux

$$\Phi_E = \int_{\text{Area}} \vec{E} \cdot d\vec{A}$$



$$\Phi_B = \int_{\text{Area}} \vec{B} \cdot d\vec{A}$$

area # field lines passing through the area.

Magnetic Gauss' Law

$$\oint \vec{B} \cdot d\vec{A} = 0$$

no magnetic monopoles observed!

- Magnetic Material

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

Contribution due to magnetic dipoles trapped in the material

Contribution from free currents

true average field

$$= \mu_0 (1 + \chi_M) \vec{H}$$

$$= \mu \vec{H}$$

Magnetic Susceptibility

- Units

  H: A/m M: A/M μ: N/A^2

- Magnetization

$$\vec{M} = \chi_m \vec{H}$$

$$= \Sigma \text{ magnetic dipoles} / Vol$$

- paramagnetism

  $0 < \chi_m << 1$ dipoles align w/ $\vec{H}$

- Diamagnetism

  $|\chi_m| << 1$ and $\chi_m < 0$

  dipoles antiparallel to $\vec{H}$

- Ferromagnetism

  Strong response to the applied field

  permanent magnets

- Induction

Faraday's Law

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

↑

Emf w/ dimensions of Volts

Minus sign means the Emf is established to oppose the change in flux. Lenz's Law

- Maxwell's Equations
  - the two Gauss's Laws
  - Faraday's Law
  - Ampère's Law